Mark Scheme (Results)

Summer 2026

Pearson Edexcel International Advanced Level
In Pure Mathematics P1
WMA11/ 01

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:
   - **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
   - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
   - **B** marks are unconditional accuracy marks (independent of M marks)

   Marks should not be subdivided.

3. Abbreviations

   These are some of the traditional marking abbreviations that will appear in the mark schemes and can be used if you are using the annotation facility on ePEN:
   - bod – benefit of doubt
   - ft – follow through
     - the symbol $\sqrt{}$ will be used for correct ft
   - cao – correct answer only
   - cso – correct solution only. There must be no errors in this part of the question to obtain this mark
   - isw – ignore subsequent working
   - awrt – answers which round to
   - SC – special case
   - oe – or equivalent (and appropriate)
   - d… or dep – dependent
   - indep – independent
   - dp – decimal places
   - sf – significant figures
   - ∗ – The answer is printed on the paper or ag- answer given
   - or d… – The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected. If you are using the annotation facility on ePEN, indicate this action by 'MR' in the body of the script.

6. If a candidate makes more than one attempt at any question:
   a) If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   b) If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Pure Mathematics Marking

(NB specific mark schemes may sometimes override these general principles)

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2+bx+c)=(x+p)(x+q)$, where $|pq|=|c|$ leading to $x = \dots$

$(ax^2+bx+c)=(mx+p)(nx+q)$, where $|pq|=|c|$ and $|mn|=|a|$ leading to $x = \dots$

2. Formula

Attempt to use correct formula (with values for a, b and c)

3. Completing the square

Solving $x^2+bx+c=0$: $(x \pm \frac{b}{2})^2 \pm q \pm c, \quad q \neq 0$ leading to $x = \dots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1 ($x^n \to x^{n-1}$)

2. Integration

Power of at least one term increased by 1 ($x^n \to x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.

Question Number	Scheme	Marks
1.	$$\int\left(3x+\frac{2}{x}\right)^2\mathrm{d}x$$	
	$$\left(3x+\frac{2}{x}\right)^2=9x^2+12+4x^{-2}$$	M1 A1
	$$\int\left(3x+\frac{2}{x}\right)^2\mathrm{d}x=\int\left(9x^2+12+4x^{-2}\right)\mathrm{d}x=3x^3+12x-\frac{4}{x}+c$$	M1 A1 A1
		(5 marks)

Notes

M1: Attempts to multiply out. Requires a minimum of $...x^2+\frac{...}{x^2}$ oe e.g. $...x^2+...x^{-2}$.

A1: For $9x^2+12+\frac{4}{x^2}$ oe e.g. $9x^2+12+4x^{-2}$ but allow this unsimplified with the terms uncollected e.g. $9x^2+6+6+\frac{4}{x^2}$ but **not** e.g. $9x^2+\frac{6x}{x}+\frac{6x}{x}+\frac{4}{x^2}$ i.e. the middle terms need to become 6's

M1: For an attempt to integrate a term in x following an attempt to expand.
It is for either $...x^2\to...x^3$ or $...x^{-2}\to...x^{-1}$ but allow the index to be unprocessed, e.g. $...x^2\to...x^{2+1}$ or $...x^{-2}\to...x^{-2+1}$
Do **not** allow for just e.g. $12\to12x$.
Condone poor notation e.g. missing integral signs and just look for the integrated terms.

A1: For at least 2 correct and simplified terms.
It is for any 2 of $3x^3$, $12x$ or $-\frac{4}{x}\left(\text{or }-4x^{-1}\right)$.
Condone $+\frac{4x^{-1}}{-1}$ for **this** mark.

A1: $3x^3+12x-\frac{4}{x}+c$ or exact simplified equivalent e.g. $3x^3+12x-4x^{-1}+c$ including the $+c$
Do **not** condone $+\frac{4x^{-1}}{-1}$ for this mark.
Ignore any spurious notation e.g. $\int\left(9x^2+12+4x^{-2}\right)\mathrm{d}x=\int3x^3+12x-\frac{4}{x}+c$
Apply isw once a correct answer is seen.

<u>Attempts to combine the terms inside the brackets first:</u>

$$\text{e.g. }\left(3x+\frac{2}{x}\right)^2=\left(\frac{3x^2+2}{x}\right)^2=\frac{9x^4+12x^2+4}{x^2}=9x^2+12+\frac{4}{x^2}$$

M1: Combines terms to obtain $\frac{...x^2+...}{x}$, squares numerator and denominator and writes as separate terms. Requires a minimum of $...x^2+\frac{...}{x^2}$ oe e.g. $...x^2+...x^{-2}$.
The rest of the marks can be applied as above.

Question Number	Scheme	Marks
2(a)	Gradient of l_2 is $\frac{1}{3}$ **or** y coordinate of P is -2	B1
	$y+2=\frac{1}{3}(x-4)$ or e.g. $\frac{y+2}{x-4}=\frac{1}{3}$ or e.g. $y=\frac{1}{3}x+c \Rightarrow -2=\frac{1}{3}\times 4+c \Rightarrow c=...\left(-\frac{10}{3}\right)$	M1
	$x-3y-10=0$	A1
		(3)

Notes

B1: States or uses gradient l_2 is $\frac{1}{3}$ or y coordinate of P is -2

M1: Correct attempt at the equation of the normal using $x = 4$, a gradient of $\frac{1}{3}$ and a value of y which has come from an attempt to substitute $x = 4$ into $y = 10 - 3x$ **with the values correctly placed**.
If they use $y = mx + c$, they must reach at least $c = \ldots$

A1: For $x-3y-10=0$ or any integer multiple of this equation in the required form.
Allow $1x-3y-10=0$ and allow e.g. $0=x-3y-10$ and allow the terms in any order.

(b)	States or uses coordinates of N as $\left(0, -\frac{10}{3}\right)$	B1ft
	Area of triangle $MPN = \frac{1}{2}\times\left(10+\frac{10}{3}\right)\times 4$ or e.g. $\frac{1}{2}MP\times NP = \frac{1}{2}\sqrt{4^2+12^2}\times\sqrt{4^2+\left(\frac{10}{3}-2\right)^2}\left(=\frac{1}{2}\times 4\sqrt{10}\times\frac{4\sqrt{10}}{3}\right)$ or e.g. $\frac{1}{2}\begin{vmatrix} 0 & 0 & 4 & 0 \\ 10 & -\frac{10}{3} & -2 & 10 \end{vmatrix} = \frac{1}{2}\left(4\times 10 - 4\times -\frac{10}{3}\right)$ which may be seen as e.g. $\frac{1}{2}\times 4\left(10-\left(-\frac{10}{3}\right)\right)+\frac{1}{2}\times 0\times\left(-\frac{10}{3}-(-2)\right)+\frac{1}{2}\times 0\times(-2-10)$ or e.g. $\frac{1}{2}\times 12\times 4+\frac{1}{2}\times\frac{4}{3}\times 4$ or e.g. $\frac{1}{2}MN\times NP\sin MNP = \frac{1}{2}\times\frac{40}{3}\times\sqrt{4^2+\left(\frac{10}{3}-2\right)^2}\sin\left\{\cos^{-1}\left(\frac{\left(\frac{4\sqrt{10}}{3}\right)^2+\left(\frac{40}{3}\right)^2-\left(4\sqrt{10}\right)^2}{2\times\frac{4\sqrt{10}}{3}\times\frac{40}{3}}\right)\right\}$	M1
	$=\frac{80}{3}$	A1
		(3)

Notes

(b)

B1ft: States or uses coordinates of N as $\left(0, -\frac{10}{3}\right)$. Follow through on their $-\frac{c}{b}$ from part (a).

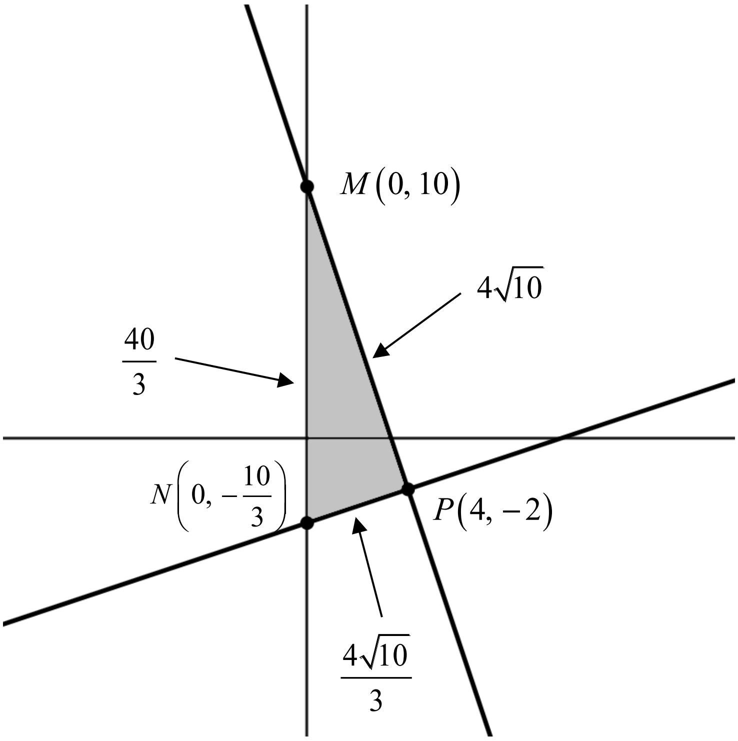
May be implied and may be seen on a diagram.

M1: Attempts area of triangle MPN using a **fully correct method** for their values.
Note that there are many different ways to find the required area so you may need to check their working carefully.
In all cases, the method must be fully correct for their values including correct use of e.g. Pythagoras or the cosine rule if they are used.
Note that M and N must both be on the y-axis so if e.g. they substitute $y = 0$ into either straight line equation and treat the resulting x values as y values, this scores M0

A1: For $\frac{80}{3}$ or **exact** equivalent e.g. $26\frac{2}{3}$ or $26.\dot{6}$ but not e.g. 26.6 or 26.7 unless the correct exact value is seen previously then apply isw. Allow recovery from inexact intermediate working provided the method is correct and the correct exact answer is obtained.
Ignore any reference to units.

		(6 marks)

Diagram for 2(b):



Question Number	Scheme	Marks
3(a)	$A=\frac{1}{2}ab\sin\theta \Rightarrow 60=\frac{1}{2}\times15\times10\times\sin\theta° \Rightarrow \sin\theta°=\frac{4}{5}$ * or e.g. $A=\frac{1}{2}\text{base}\times\text{height}=60 \Rightarrow 60=\frac{1}{2}\times15\times h=60 \Rightarrow h=8 \Rightarrow \sin\theta=\frac{8}{10}=\frac{4}{5}$ or e.g. by verification $A=\frac{1}{2}ab\sin\theta=\frac{1}{2}\times15\times10\times\frac{4}{5}=60 \Rightarrow \sin\theta°=\frac{4}{5}$	B1*

Notes

Condone use of a different letter in (a) e.g. $\frac{1}{2}\times15\times10\times\sin C \Rightarrow \sin C=\frac{4}{5}$

B1*: Score for a correct equation using the area is 60, e.g. $60=\frac{1}{2}\times15\times10\times\sin\theta$ followed by $\sin\theta=\frac{4}{5}$ with no incorrect work seen or uses $A=\frac{1}{2}\text{base}\times\text{height}=60$ to deduce the height is 8 and hence $\sin\theta=\frac{4}{5}$.

Condone

The alternative by verification requires showing that $\frac{1}{2}\times15\times10\times\frac{4}{5}=60$ followed by a (minimal) conclusion e.g. so $\sin\theta°=\frac{4}{5}$, QED, ✓ etc. with no incorrect work seen.

Note that any reference to the value of θ can be ignored as long as the rest of the working is correct e.g. allow $60=\frac{1}{2}\times15\times10\times\sin\theta° \Rightarrow \theta=53.1...° \Rightarrow \sin\theta°=\frac{4}{5}$

Ignore any units seen e.g. $A=\frac{1}{2}ab\sin\theta \Rightarrow 60=\frac{1}{2}\times15\times10\times\sin\theta° \Rightarrow \sin\theta°=\frac{4}{5}$ radians.

(b)	Deduces that $(\theta =)180-53.1=\text{awrt } 127$	B1
	e.g. $\left(BC^2\right)=10^2+15^2-2\times10\times15\cos126.9°$	M1
	$\Rightarrow BC=\sqrt{505}$ awrt 22.5 (m)	A1
		(3)

Notes

B1: Deduces that $(\theta =)180-53.1=\text{awrt}127$ or uses $\cos\theta=-\frac{3}{5}$ which may be implied.

May be implied by the sight or use of awrt 127

M1: Attempts to use the cosine rule (condone the omission of the lhs) to find BC or BC^2

Must be using either the obtuse angle

$\theta=180-\sin^{-1}\frac{4}{5}\left(\text{or }\cos\theta=-\frac{3}{5}\right)$ and a correct formula with the sides in the correct places

or the acute angle: $\theta=\sin^{-1}\frac{4}{5}\left(\text{or }\cos\theta=\frac{3}{5}\right)$ and a correct formula with the sides in the correct places.

For reference, use of the acute angle leads to $BC=\sqrt{145}\left(=12.04...\right)$

If they use e.g. $BC=\sqrt{10^2+15^2+2\times10\times15\cos53.1°}$ this is correct for the obtuse angle.

But do not allow if this follows e.g. $BC=\sqrt{a^2+b^2+2ab\cos C}$

Note that incorrectly using $\theta=\frac{4}{5}$ e.g. $\left(BC^2\right)=10^2+15^2-2\times10\times15\cos\left(\frac{4}{5}\right)$ scores M0

A1: Length BC is awrt 22.5 (m) but allow exact value of $\sqrt{505}$ (m)

Units are **not** required but if any are given they must be correct.

Allow the use of radians to score for equivalent work in (b) e.g.

$$\theta=\pi-0.927...=\text{awrt }2.21\Rightarrow\left(BC^2\right)=10^2+15^2-2\times10\times15\cos2.21\Rightarrow BC=22.5$$

(c)	$\dfrac{\sin ABC}{10} = \dfrac{\sin '126.9°'}{'22.5'} \Rightarrow \sin ABC = ...$ or e.g. $60 = \dfrac{1}{2} \times 15 \times '22.5' \times \sin ABC \Rightarrow \sin ABC = ...$ or e.g. $10^2 = 15^2 + '505' - 2 \times 15\sqrt{'505'} \cos ABC \Rightarrow \cos ABC = ...$	M1
	Smallest angle (ABC) = awrt $20.9°$ (or awrt $20.8°$)	A1
		(2)

Notes:

Allow equivalent work in radians but the final answer must be in degrees.

M1: Full attempt to find sin ABC or cos ABC using angle $BAC = \sin^{-1}\dfrac{4}{5}$ which may be acute or obtuse and follow through their answer to part (b).

Score for $\dfrac{\sin ABC}{10} = \dfrac{\sin '126.9°'(\text{or } '53.1°')}{'22.5'}\left(\text{or e.g. } \dfrac{4/5}{'22.5'}\right) \Rightarrow \sin ABC = ...$

or $60 = \dfrac{1}{2} \times 15 \times '22.5' \times \sin ABC \Rightarrow \sin ABC = ...$

or $10^2 = 15^2 + '505' - 2 \times 15'\sqrt{505}'\cos ABC \Rightarrow \cos ABC = ...$

Note that incorrectly using $\theta = \dfrac{4}{5}$ e.g. $\dfrac{\sin ABC}{10} = \dfrac{\sin\left(\frac{4}{5}\right)}{'22.5'} \Rightarrow \sin ABC = ...$ scores M0

Allow alternative methods that correctly find angle ACB first and then $ABC = 180 - \sin^{-1}\dfrac{4}{5} - ACB$

e.g. $60 = \dfrac{1}{2} \times 10 \times '22.5' \times \sin ACB \Rightarrow ACB = ...(32.2...) \Rightarrow ABC = 180 - \sin^{-1}\dfrac{4}{5} - '32.2'$

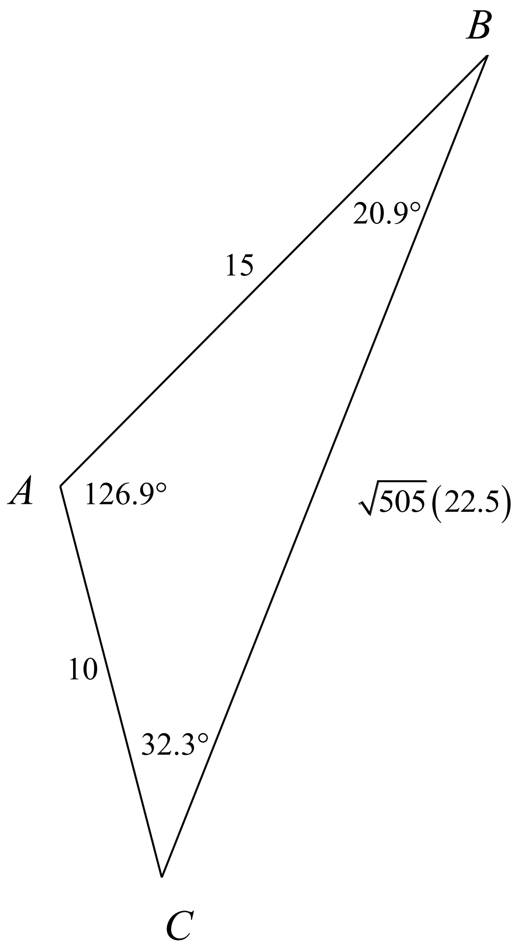
or e.g. $\dfrac{\sin ACB}{15} = \dfrac{\sin '126.9°'(\text{or } '53.1°')}{'22.5'} \Rightarrow ACB = ...(32.2...) \Rightarrow ABC = 180 - \sin^{-1}\dfrac{4}{5} - '32.2'$

A1: Awrt $20.9°$ or awrt $20.8°$ following correct work in (b) e.g. do not allow this mark if the 22.5 was obtained fortuitously in part (b). (Unless they start again).
The degrees symbol is **not** required.
If both other angles have been found then the awrt $20.9°$ or awrt $20.8°$ must clearly be selected.

For reference use of the acute angle in (a) leading to $BC = \sqrt{145}$ in (b) gives an answer of $41.6...°$ in (c)

		(6 marks)

Diagram for Q3:



Question Number	Scheme	Marks
4(a)	$\dfrac{\mathrm{d}\left(x^3\right)}{\mathrm{d}x}=3x^2$	B1
	$\dfrac{\mathrm{d}\left(2x^2\sqrt{x}\right)}{\mathrm{d}x}=\dfrac{\mathrm{d}\left(2x^{\frac{5}{2}}\right)}{\mathrm{d}x}=...x^{\frac{3}{2}}$	M1
	$\left(\mathrm{f}'(x)=\right)3x^2-5x^{\frac{3}{2}}$	A1
	$3x^2-5x^{\frac{3}{2}}=0\Rightarrow x^{\frac{3}{2}}\left(3x^{\frac{1}{2}}-5\right)=0\Rightarrow 3x^{\frac{1}{2}}-5=0$ or e.g. $3x^2-5x^{\frac{3}{2}}=0\Rightarrow 3x^{\frac{1}{2}}-5=0$ or e.g. $3x^2-5x^{\frac{3}{2}}=0\Rightarrow 3x^2=5x^{\frac{3}{2}}\Rightarrow 9x^4=25x^3\Rightarrow 9x=25\left(\text{or } x=\dfrac{25}{9}\right)$	**d**M1
	$x=\dfrac{25}{9}$	A1 cso
		(5)

Notes

Mark (a) and (b) together e.g. ignore labelling in this question.

B1: Correct differentiation of x^3. Allow the index to be unprocessed e.g. $3x^{3-1}$.

M1: Differentiates $2x^2\sqrt{x}$ to obtain $...x^{\frac{3}{2}}$. Allow the index to be unprocessed e.g. $...x^{\frac{5}{2}-1}$

May be implied by an attempt to use the product rule e.g.

$\left(2x^2\sqrt{x}\right)'=\frac{1}{2}\times 2x^2\times x^{-\frac{1}{2}}+4x\times x^{\frac{1}{2}}=\frac{1}{2}\times 2x^{\frac{3}{2}}+4x^{\frac{3}{2}}$

Allow for $\left(2x^2\sqrt{x}\right)'=\alpha x^{\frac{3}{2}}+\beta x^{\frac{3}{2}}$ and allow the indices to be unprocessed e.g. $\alpha x^{2-\frac{1}{2}}+\beta x^{1+\frac{1}{2}}$

A1: $\left(\mathrm{f}'(x)=\right)3x^2-5x^{\frac{3}{2}}$ oe. May be unsimplified but indices must now be processed and terms combined if necessary. Condone poor notation e.g. spurious integral signs.

e.g. allow $\left(\mathrm{f}'(x)=\right)3x^2-\frac{5}{2}\times 2x^{\frac{3}{2}}$ and allow $\left(\mathrm{f}'(x)=\right)3x^2-5x^{\frac{3}{2}}+0$ but do not allow

$\left(\mathrm{f}'(x)=\right)3x^2-5x^{\frac{3}{2}}+c$. "$\mathrm{f}'(x)=$" is not required.

dM1: Sets $\mathrm{f}'(x)=0$ where $\mathrm{f}'(x)$ is of the form $px^2-qx^{\frac{3}{2}}$, $p,q>0$ and **uses correct index work** to obtain $ax^b=c,\ ac>0$ or equivalent e.g. $ax^b-c=0,\ ac>0$ where a and/or b could be 1

but they cannot go from just $3x^2-5x^{\frac{3}{2}}=0\Rightarrow x=\dfrac{25}{9}$ as the question asks for all stages of working to be shown and a calculator not to be used.

See main scheme for some examples and the next page for further examples of acceptable work.

Depends on the first method mark.

A1: $x=\dfrac{25}{9}$ following the award of all previous marks.

There must be no other solutions so if e.g. $x=0$ is given and not rejected, score A0

<u>Further examples of acceptable work for the dM1 in part (a):</u>

$$3x^2-5x^{\frac{3}{2}}=0 \Rightarrow 3-5x^{-\frac{1}{2}}=0$$

$$y=\sqrt{x} \Rightarrow 3y^4-5y^3=0 \Rightarrow y=\frac{5}{3}\left(\text{or } \ 3y-5=0\right)$$

Condone: $3x^2-5x^{\frac{3}{2}}=0 \Rightarrow 9x^4-25x^3=0 \Rightarrow x=\frac{25}{9}$

(b)(i)	$\left(\mathrm{f}''(x)=\right)6x-\dfrac{15}{2}x^{\frac{1}{2}}$	M1 A1
	$6x-\dfrac{15}{2}x^{\frac{1}{2}}=66 \Rightarrow \quad 4x-5x^{\frac{1}{2}}-44=0$ *	A1*

Notes

M1: Differentiates an expression of the form px^2+qx^r to obtain $...x-...x^{r-1}$ where r is a non-integer fraction. Condone poor notation e.g. spurious integral signs.

A1: Correct differentiation $\left(\mathrm{f}''(x)=\right)6x-\dfrac{15}{2}x^{\frac{1}{2}}$. May be unsimplified but indices must be processed.

e.g. allow $\left(\mathrm{f}'(x)=\right)6x-\dfrac{3}{2}\times 5x^{\frac{1}{2}}$. Condone poor notation and just look for the correct expression. $"\mathrm{f}''(x)="$ is not required.

A1*: Sets $6x-\dfrac{15}{2}x^{\frac{1}{2}}=66$ and reaches $4x-5x^{\frac{1}{2}}-44=0$ from fully correct work.

An intermediate step is **not** required.

(b)(ii)	$$4x-5x^{\frac{1}{2}}-44=\left(x^{\frac{1}{2}}-4\right)\left(4x^{\frac{1}{2}}+11\right)$$ or e.g. $$4x-5x^{\frac{1}{2}}-44=0\Rightarrow\left(\sqrt{x}=\right)\frac{5\pm\sqrt{5^2-4(4)(-44)}}{8}\left(=4,-\frac{11}{4}\right)$$	M1
	e.g. $x^{\frac{1}{2}}-4=0$ only as $x^{\frac{1}{2}}$ cannot be < 0 so x = 16 or e.g. If $x^{\frac{1}{2}}=-\frac{11}{4}$, $x=\frac{121}{16}\Rightarrow 4x-5x^{\frac{1}{2}}-44=-\frac{55}{2}\neq 0$ If $x^{\frac{1}{2}}=4, x=16\Rightarrow 4x-5x^{\frac{1}{2}}-44=0$ x = 16	A1*
		(5)

Notes

M1: For an attempt to **factorise** $4x-5x^{\frac{1}{2}}-44$ **or** a non-calculator attempt to **solve** $4x-5x^{\frac{1}{2}}-44=0$
Attempts to factorise or solve must follow the usual rules (see general guidance)
Do **not** condone attempts to solve/factorise a different equation.
Do not be concerned at this stage if it is unclear whether their values represent e.g. x rather than $\sqrt{x}$. If the quadratic formula is used look for a minimum of $\left(\sqrt{x}=\right)\frac{5\pm\sqrt{5^2-4(4)(-44)}}{8}$ or $\left(\sqrt{x}=\right)\frac{5\pm\sqrt{729}}{8}$ or $\left(\sqrt{x}=\right)\frac{5\pm 27}{8}$

May be implied by substitution e.g. $y=\sqrt{x}\Rightarrow 4x-5x^{\frac{1}{2}}-44=4y^2-5y-44=(y-4)(4y+11)$

or even $x=\sqrt{x}\Rightarrow 4x-5x^{\frac{1}{2}}-44=4x^2-5x-44=(x-4)(4x+11)$

If answers are **just written down**, this scores M0 e.g. $\left(\sqrt{x}=\right)\frac{-b\pm\sqrt{b^2-4ac}}{2a}=4, -\frac{11}{4}$

A1*: Must follow M1 and requires:

- Correct factorisation or correct values of $\sqrt{x}$
- **Either** a valid reason to reject $4x^{\frac{1}{2}}+11=0$ e.g. because $x^{\frac{1}{2}}<0$
  or a valid reason that $x^{\frac{1}{2}}-4=0$ gives the only solution e.g. because $x^{\frac{1}{2}}>0$
- A conclusion that x = 16

Allow as a minimum e.g. $\sqrt{x}=-\frac{11}{4}$ (which is) **< 0** (so reject) so x = 16

Do not allow incorrect arguments such as "$\sqrt{x}\neq-\frac{11}{4}$ as you can't square root a negative number"

or insufficient reasoning e.g. $\sqrt{x}=-\frac{11}{4}$ (reject)

An alternative for the A1* would be to check which values of *x* are valid e.g.

$x=\frac{121}{16}\Rightarrow 4\left(\frac{121}{16}\right)-5\left(\frac{11}{4}\right)-44=-\frac{55}{2}$

$x=16\Rightarrow 4(16)-5(4)-44=0$

x = 16 is the only solution

This must follow M1 and requires:

- Correct values of x
- Correct evaluation of $4x-5x^{\frac{1}{2}}-44$ for $x=\frac{121}{16}$ and $x=16$
- A (minimal) conclusion e.g. x = 16, QED, tick etc.

(10 marks)

Question Number	Scheme	Marks
5(a)	$-2x^2+12x+7 \equiv -2(x\pm...)^2+...$	B1
	$\equiv -2(x-3)^2+25$	M1 A1
		(3)
(b)	$(3, 25)$	M1 A1
		(2)

Notes

(a)

B1: States that $a=-2$ or else writes $-2x^2+12x+7$ in the form $-2(x\pm\alpha)^2+\beta,\ \alpha,\beta\neq 0$ or e.g. $-2\left[(x\pm\alpha)^2+\beta\right],\ \alpha,\beta\neq 0$

M1: For $-2x^2+12x+7\equiv -2\left(x\pm 3\right)^2\pm k$ or e.g. $-2\left[\left(x\pm 3\right)^2\pm k\right],\ k\neq 0$

A1: Correct values of a, b and c or writes $-2x^2+12x+7$ as $-2(x-3)^2+25$

<u>Special case – candidates who change the sign of the expression:</u>

e.g. $-2x^2+12x+7\to 2x^2-12x-7\equiv 2\left(x-3\right)^2-25$

Can score a maximum of B0M1A0 if they obtain $2\left(x\pm 3\right)^2\pm k, k\neq 0$ or equivalent.

If the signs are subsequently reversed then the main scheme can be applied and all marks are available.

(b) **May be seen on the Figure or within the body of the question.**

Note that this is an independent part of the question and not a "Hence…"

As such they can either score the method mark (and possibly the A mark) as described below if they use their completed square form from part (a) or they can answer it in isolation and score M1 for one correct coordinate and A1 for both correct coordinates.

M1: Award for **either** $x = 3$ or $y = 25$ **or** follow through their $-2(x-3)^2+25$ so for their $a(x+b)^2+c$ award for $x=-b$ or $y=c$

Note that x may be found from $y=-2x^2+12x+7\Rightarrow \dfrac{dy}{dx}=-4x+12,\ -4x+12=0\Rightarrow x=3$

or $y=-2x^2+12x+7\Rightarrow$ vertex is at $x=-\dfrac{12}{2\times -2}=3$

May be seen as $x = \ldots$ or $y = \ldots$ or as part of a coordinate pair.

A1: $(3, 25)$ Condone missing brackets e.g. $3, 25$ or unusual punctuation e.g. $(3;25)$ or a vector e.g. $\begin{pmatrix}3\\25\end{pmatrix}$ Do **not** allow coordinates the wrong way round unless the correct answer is seen previously then apply isw. This mark is **not** follow through so the coordinates must be correct. Correct answer only scores both marks.

(c)	$y \leqslant -2x^2 + 12x + 7$, $y \geqslant "\frac{25}{3}"x$, $x \geqslant 0$	B1ft B1ft
		(2)
		(7 marks)

(c) Notes

Allow use of $f(x)$ for y.

Do not allow use of R for y e.g. $R \leqslant -2x^2 + 12x + 7$ or e.g. $y >$ line

B1ft: Any 2 of $y \leqslant -2x^2 + 12x + 7$, $y \geqslant "\frac{25}{3}"x$, $x \geqslant 0$ with inequalities in the correct directions and ignore any other inequalities for this mark.

Allow strict or non-strict inequalities and condone a mix of both for this mark.

Follow through their equation for l $\left(y = \frac{c}{-b}\right)$ or their coordinates from part (b) and their possibly incorrect completed square form for C.

Examples:

$$y \leqslant -2x^2 + 12x + 7, \quad y \geqslant "\frac{25}{3}"x$$

$$"\frac{25}{3}"x < y < -2x^2 + 12x + 7$$

$$y \leqslant -2x^2 + 12x + 7, \ x > 0$$

For the restriction on x, allow $0 < x < a$ where $a \geqslant "3"$

B1ft: Correct and fully **consistent** inequalities and no incorrect inequalities.

Follow through their equation for l $\left(y = \frac{c}{-b}\right)$ or their coordinates from part (b) and their possibly incorrect completed square form for C.

For the restriction on x, allow $0 < x < a$ where $a \geqslant "3"$ but must be consistent with the other inequalities i.e. strict or non-strict and must not contradict the inequalities given for the line and the curve if "3" is the upper limit.

e.g. $"\frac{25}{3}x" \leqslant y \leqslant -2x^2 + 12x + 7, \ \ 0 \leqslant x < 3$ scores B1ftB0ft

If extra inequalities are given that do not contradict then they can be ignored (see examples below)

<u>Examples of full marks in (c):</u>

$$y \leqslant -2x^2 + 12x + 7, \quad y \geqslant "\frac{25}{3}x", \quad x \geqslant 0$$

$$"\frac{25}{3}x" < y < -2x^2 + 12x + 7, \quad 0 < x < 3$$

$$"\frac{25}{3}x" < y, \ y < -2x^2 + 12x + 7, \quad 0 < x, \ \ y > 0$$

Question Number	Scheme	Marks
6(a)	States or implies that $\left(\mathrm{f}(x)=\right)k\left(2x\pm3\right)^2\left(x\pm4\right)$	M1
	$-12=k\left(2\left(0\right)+3\right)^2\left(0-4\right)\Rightarrow k=...$	**d**M1
	$\left(\mathrm{f}(x)=\right)\frac{1}{3}\left(2x+3\right)^2\left(x-4\right)$ oe e.g. $\left(\mathrm{f}(x)=\right)\frac{1}{3}\left(4x^3-4x^2-39x-36\right)$	A1
		(3)

Notes

In part (a) "f(*x*) =" is not required so just look for the correct expression.

The correct <u>expression</u> must be seen not just e.g. $\left(\mathrm{f}(x)=\right)k\left(2x+3\right)^2\left(x-4\right)$ followed by $k=\frac{1}{3}$ but allow if the correct expression subsequently appears in part (b) provided they have not given an incorrect expression in part (a).

A fully correct expression in (a) with no working scores M1dM1A1

M1: States or implies that $\left(\mathrm{f}(x)=\right)k\left(2x\pm3\right)^2\left(x\pm4\right)$ o.e. such as $\left(\mathrm{f}(x)=\right)\beta\left(x\pm\frac{3}{2}\right)^2\left(x\pm4\right)$

Allow β or k to be any non-zero constant including 1 and could be any letter.

You can ignore a spurious "= 0" e.g. $\left(\mathrm{f}(x)=\right)k\left(2x\pm3\right)^2\left(x\pm4\right)=0$ or e.g. $\left(\mathrm{f}(x)=\right)k\left(2x\pm3\right)^2\left(x\pm4\right)=-12$ for this mark.

Note that e.g. $\left(\mathrm{f}(x)=\right)k\left(3\pm2x\right)^2\left(4\pm x\right)$ or e.g. $\left(\mathrm{f}(x)=\right)\beta\left(\frac{3}{2}\pm x\right)^2\left(4\pm x\right)$ are also acceptable for this mark.

dM1: Substitutes $x = 0$ and $y = -12$ into $\left(\mathrm{f}(x)=\right)k\left(2x\pm3\right)^2\left(x\pm4\right)$ to find a value for k.

Alternatively substitutes $x = 0$ and $y = -12$ into $\left(\mathrm{f}(x)=\right)\beta\left(x\pm\frac{3}{2}\right)^2\left(x\pm4\right)$ to find a value for β.

If no substitution is seen, the coefficient must be correct for their function.

Note that e.g. $\left(\mathrm{f}(x)=\right)k\left(3\pm2x\right)^2\left(4\pm x\right)$ or e.g. $\left(\mathrm{f}(x)=\right)\beta\left(\frac{3}{2}\pm x\right)^2\left(4\pm x\right)$ are also acceptable for this mark.

Depends on the first method mark.

A1: $\left(\mathrm{f}(x)=\right)\frac{1}{3}\left(2x+3\right)^2\left(x-4\right)$ o.e. e.g. $\left(\mathrm{f}(x)=\right)\frac{4}{3}\left(x+\frac{3}{2}\right)^2\left(x-4\right)$ or $\left(\mathrm{f}(x)=\right)-\frac{1}{3}\left(2x+3\right)^2\left(4-x\right)$

Condone a spurious "= 0" e.g. $\left(\mathrm{f}(x)=\right)\frac{1}{3}\left(2x+3\right)^2\left(x-4\right)=0$

Apply isw once a correct expression is seen.

Note that if they start with a **correct** expression e.g. $\left(\mathrm{f}(x)=\right)k\left(2x+3\right)^2\left(x-4\right)$ and expand at the dM1 stage and correctly reach e.g. $-36k=-12\rightarrow k=\frac{1}{3}\rightarrow\left(\mathrm{f}(x)=\right)\frac{1}{3}\left(2x+3\right)^2\left(x-4\right)$ then any errors in the non-constant terms can be ignored and full marks awarded if correct.

	(a) Alternative by simultaneous equations:	
	$\left(\mathrm{f}(x)=\right)ax^3+bx^2+cx+d$ $(0,\ -12)\rightarrow d=-12$ $\left(-\frac{3}{2},\ 0\right)\rightarrow 0=-\frac{27}{8}a+\frac{9}{4}b-\frac{3}{2}c-12$ $(4,\ 0)\rightarrow 0=64a+16b+4c-12$ $\mathrm{f}'(x)=3ax^2+2bx+c\rightarrow 0=\frac{27}{4}a-3b+c$	M1
	$a=\frac{4}{3},\ b=-\frac{4}{3},\ c=-13\rightarrow\left(\mathrm{f}(x)=\right)\frac{4}{3}x^3-\frac{4}{3}x^2-13x-12$	**d**M1A1

Notes

M1: Starts with a general cubic and uses the given information to produce 3 equations in 3 unknowns with $d=\pm 12$

dM1: Solves to find all unknowns. You do not need to check the working as long as all values are found. They may be found using a calculator.

A1: $\left(\mathrm{f}(x)=\right)\frac{4}{3}x^3-\frac{4}{3}x^2-13x-12$ oe. Condone a spurious "= 0" e.g. $\left(\mathrm{f}(x)=\right)\frac{4}{3}x^3-\frac{4}{3}x^2-13x-12=0$
Apply isw once a correct answer is seen.

(b)	$\frac{1}{3}(2x+3)^2(x-4)=2x-12 \Rightarrow (4x^2+12x+9)(x-4)=6x-36$ $\Rightarrow 4x^3-16x^2+12x^2-48x+9x-36=6x-36$ $\Rightarrow 4x^3-4x^2=45x$ or e.g. $4x^3-4x^2-45x=0$	M1A1
	$x(4x^2-4x-45)=0 \Rightarrow x=\frac{4\pm\sqrt{736}}{8}$	**d**M1
	$=\frac{1\pm\sqrt{46}}{2}$	A1
		(4)
		(7 marks)

(b) Notes

(b)

M1: Sets their $f(x)$ **which must be a cubic function** equal to $2x-12$ and starts to simplify by expanding the lhs and collecting terms on lhs or from rhs to lhs.
Note that the lhs may already be expanded e.g. if the alternative was used in part (a).

A1: Correct simplified 3 term equation $4x^3-4x^2-45x=0$ or $4x^2-4x-45=0$
oe e.g. $\frac{4}{3}x^3-\frac{4}{3}x^2-15x=0$ or $\frac{4}{3}x^2-\frac{4}{3}x-15=0$
The terms do not necessarily need to be all on one side e.g. $\frac{4}{3}x^2-\frac{4}{3}x=15$ is acceptable.
The "= 0" may be implied by their attempt to solve.

dM1: Solves the resulting 3 term equation which must either be a 3TQ or a 3 term cubic equation with no constant term.
The solution of their equation can be by any method including a calculator.
If values are just written down you may need to check.
Note that it is acceptable to go from e.g. $4x^3-4x^2-45x=0$ to the correct roots using a calculator.
Note that a cubic equation of the required form may have been obtained fortuitously but this mark is still available e.g. if they obtained $(f(x)=)(2x+3)^2(x-4)+24$ in part (a).

A1: Correct values $\frac{1\pm\sqrt{46}}{2}$ oe e.g. $\frac{1}{2}\pm\frac{\sqrt{46}}{2}$
Condone the inclusion of $x = 0$
Ignore any attempts to find the y values.

Question Number	Scheme	Marks
7(a)	$\frac{1}{2}r^2\theta = 1$ **or** $2r + r\theta = 8$	B1
	$\frac{1}{2}r^2\theta = 1$ **and** $2r + r\theta = 8$	B1
		(2)

Notes

Mark (a) and (b) together i.e. ignore any labelling.

Note that e.g. $\frac{\theta}{360} \times \pi r^2 = 1$ is incorrect for the area equation.

B1: Either $\frac{1}{2}r^2\theta = 1$ **or** $2r + r\theta = 8$ or their exact equivalents e.g. $\frac{1}{2} \times r^2\theta = 1$ or $2 \times r + r \times \theta = 8$

Ignore any units included in the equations e.g. $\frac{1}{2}r^2\theta = 1\text{cm}^2$ or e.g. $2r + r\theta = 8\text{cm}$

B1: Both $\frac{1}{2}r^2\theta = 1$ **and** $2r + r\theta = 8$ or their exact equivalents e.g. $\frac{1}{2} \times r^2\theta = 1$ or $2 \times r + r \times \theta = 8$

Ignore any units included in the equations e.g. $\frac{1}{2}r^2\theta = 1\text{cm}^2$ or e.g. $2r + r\theta = 8\text{cm}$

Note that these equations may be seen in the body of the question and (less likely) may be implied if they answer (b) without writing them down explicitly e.g. $2r + r\left(\frac{2}{r^2}\right) = 8$ or

$\frac{1}{2}\left(\frac{8}{2+\theta}\right)^2\theta = 1$

(b)	$\frac{1}{2}r^2\theta = 1 \Rightarrow \theta = \frac{2}{r^2},\ 2r + \frac{2}{r} = 8$ or $2r + r\theta = 8 \Rightarrow r = \frac{8}{2+\theta},\ \frac{1}{2}r^2\theta = 1 \Rightarrow \frac{1}{2}\left(\frac{8}{2+\theta}\right)^2\theta = 1$	M1
	$r^2 - 4r + 1 = 0$ oe or $\theta^2 - 28\theta + 4 = 0$ oe	A1
	e.g. $(r-2)^2 = 3 \Rightarrow r =$ or e.g. $\theta = \frac{28 \pm \sqrt{28^2 - 4\times1\times4}}{2}$	**d**M1
	$\theta = \frac{2}{r^2} = \frac{2}{\left(2+\sqrt{3}\right)^2} = \frac{2}{7+4\sqrt{3}} = \frac{2}{7+4\sqrt{3}} \times \frac{7-4\sqrt{3}}{7-4\sqrt{3}} = ...\left(14-8\sqrt{3}\right)$ or e.g. $r = \frac{8}{2+\theta} = \frac{8}{2+14-8\sqrt{3}} = \frac{8\left(16+8\sqrt{3}\right)}{\left(16-8\sqrt{3}\right)\left(16+8\sqrt{3}\right)} = ...\left(2+\sqrt{3}\right)$	**dd**M1
	$r = 2+\sqrt{3},\ \theta = 14-8\sqrt{3}$	A1
		(5)

M1: From 2 equations in r and θ, attempts to create an equation in a single variable.
The equations must be dimensionally correct but note that **this** mark is available even if the resulting equation in one variable is linear e.g. from $\frac{1}{2}r^2\theta = 1$ and $r\theta = 8$
You do not need to be concerned about the mechanics as long as they reach an equation in one variable.

A1: Any correct 3 term quadratic equation in a single variable with terms collected but not necessarily all on one side e.g. $2r^2 - 8r + 2 = 0$ o.e e.g. $r^2 - 4r = -1$ or $\theta^2 - 28\theta + 4 = 0$ oe e.g. $\theta^2 - 28\theta = -4$ The "= 0" may be implied by their attempt to solve.
May be seen embedded e.g. $r\left(2r^2 - 8r + 2\right) = 0$

dM1: Solves their 3TQ in r or θ using a **non-calculator** method to produce a surd solution.
See General Guidance for solving a 3TQ.
Note that just quoting the quadratic formula followed by values scores M0 but that e.g. $\theta = \frac{28 \pm \sqrt{28^2 - 4\times1\times4}}{2} = 14 \pm 8\sqrt{3}$ or e.g. $r = \frac{4 \pm \sqrt{4^2 - 4\times1\times1}}{2} = 2 \pm \sqrt{3}$ is sufficient.

ddM1: Dependent upon the previous M, it is for a correct method to find the other variable from a fraction of the form $\frac{...}{a+b\sqrt{c}}$ where a, b and c are non-zero and $\sqrt{c}$ is irrational using a **non-calculator** method. E.g. multiplies numerator and denominator by the conjugate of the denominator. Note that having found θ, any progress to find r using $r = \sqrt{\frac{2}{\theta}}$ is unlikely as they will have to use a non-calculator method. This mark is for finding r or θ using a correct method and not for finding a value for e.g. r^2.

A1: $r = 2+\sqrt{3},\ \theta = 14-8\sqrt{3}$ o.e. such as $r = 2+\sqrt{3},\ \theta = 14-2\sqrt{48}$
Condone the omission of "r =" and/or "θ =" if it is clear which is which.
Note that once the rationalisation of the denominator process is shown e.g. $\frac{2}{7+4\sqrt{3}} \times \frac{7-4\sqrt{3}}{7-4\sqrt{3}}$ or $\frac{8\left(16+8\sqrt{3}\right)}{\left(16-8\sqrt{3}\right)\left(16+8\sqrt{3}\right)}$, it is acceptable to write down the final answers.
If there are any extra incorrect values then withhold this mark.

		(7 marks)

Note that in 7(b), some candidates solve simultaneously using $r=\sqrt{\frac{2}{\theta}}$ e.g.

$$\frac{1}{2}r^2\theta=1\Rightarrow r=\sqrt{\frac{2}{\theta}}\Rightarrow\theta\sqrt{\frac{2}{\theta}}+2\sqrt{\frac{2}{\theta}}=8$$

This scores M1 for an equation in one variable and then A1 for

$$\theta\sqrt{\frac{2}{\theta}}+2\sqrt{\frac{2}{\theta}}=8\Rightarrow\sqrt{2}\theta-8\sqrt{\theta}+2\sqrt{2}=0 \text{ oe}$$

i.e. a correct quadratic equation in $\sqrt{\theta}$

They will need to solve this for $\sqrt{\theta}$ using a non-calculator method e.g.

$$\sqrt{2}\theta-8\sqrt{\theta}+2\sqrt{2}=0\Rightarrow\sqrt{\theta}=\frac{8\pm\sqrt{64-16}}{2\sqrt{2}}=\frac{8\pm4\sqrt{3}}{2\sqrt{2}}$$

and then square to obtain a value for θ for **d**M1

$$\theta=\left(\frac{8\pm4\sqrt{3}}{2\sqrt{2}}\right)^2=14\pm8\sqrt{3}$$

and then **dd**M1A1 as in the main scheme.

Question Number	Scheme	Marks
8(a)(i)	$x=\frac{1}{4}\Rightarrow 10+\frac{5\sqrt{x}-4}{2x^2}=10+\frac{5\sqrt{\frac{1}{4}}-4}{2\times\left(\frac{1}{4}\right)^2}=-2$	M1 A1
(ii)	Uses their $k=-2$ and $x=\frac{1}{4}$ in $y=kx+2\ \Rightarrow y=\frac{3}{2}$	M1 A1
	$y-"\frac{3}{2}"=-\frac{1}{"-2"}\left(x-\frac{1}{4}\right)\Rightarrow y=\frac{1}{2}x+\frac{11}{8}$ or e.g. $\frac{y-"\frac{3}{2}"}{\left(x-\frac{1}{4}\right)}=-\frac{1}{"-2"}\Rightarrow y=\frac{1}{2}x+\frac{11}{8}$ or e.g. $y=-\frac{1}{"-2"}x+c\Rightarrow "\frac{3}{2}"=\frac{1}{2}\times\frac{1}{4}+c\Rightarrow c=...\left(\frac{11}{8}\right)\Rightarrow y=\frac{1}{2}x+\frac{11}{8}$	**d**M1 A1
		(6)

Notes

Mark (a) and (b) together i.e. ignore labelling.

(a)(i)

M1: Attempts to substitute $x=\frac{1}{4}$ into $\mathrm{f}'(x)=10+\frac{5\sqrt{x}-4}{2x^2}$

Condone a miscopy of the function as long as the intention is clear e.g. use of $\frac{10+5\sqrt{x}-4}{2x^2}$

A1: $(k=)-2$ The "$k=$" is not required so just look for the correct value and apply isw if they subsequently think that $y=$ their k i.e. they think they have found $\mathrm{f}\left(\frac{1}{4}\right)$ rather than $\mathrm{f}'\left(\frac{1}{4}\right)$

This mark is for evaluating $10+\frac{\frac{5}{2}-4}{2\times\left(\frac{1}{4}\right)^2}$ to give -2 regardless of what they do with it.

The correct value with no working scores both marks.

(a)(ii)

M1: Uses their value of k (however obtained) and $x=\frac{1}{4}$ in $y=kx+2$ to find y.

May be implied by their value of y but if no method is shown and their value of y is incorrect then score M0

A1: $y=\frac{3}{2}$ oe

dM1: Correct method of finding the equation of the normal using the negative reciprocal of their k with $x=\frac{1}{4}$ and their value of y. If they use "$y=mx+c$" they must reach a value for c.

It is dependent upon the previous M.

A1: $y=\frac{1}{2}x+\frac{11}{8}$ or exact equivalent but must be in the form $y=mx+c$

(b)	$\left(f'(x)=\right)10+\frac{5}{2}x^{-\frac{3}{2}}-2x^{-2} \Rightarrow \left(f(x)=\right)10x-5x^{-\frac{1}{2}}+2x^{-1}\left(+c\right)$	M1 A1 A1
	Uses $P\left(\frac{1}{4},'\frac{3}{2}'\right) \Rightarrow \frac{3}{2}=\frac{5}{2}-5\times 2+2\times 4+c \Rightarrow c=...$	**d**M1
	$\left(f(x)=\right)10x-5x^{-\frac{1}{2}}+2x^{-1}+1$	A1
		(5)

Notes

Condone spurious notation for the A marks e.g. floating integral signs and/or d*x*'s

M1: Attempts to split up the fractional term (into two terms) then integrates with two terms (from the three) having a correct index **after integrating**.

It is for 2 of, $10 \to 10x$, $\frac{5\sqrt{x}}{2x^2} \to ...x^{-\frac{3}{2}} \to ...x^{-\frac{1}{2}}$, $\frac{4}{2x^2} \to ...x^{-2} \to ...x^{-1}$

Allow the indices to be unprocessed.

Condone the omission of the + c

A1: Two correct terms of $\left(f(x)=\right)10x-5x^{-\frac{1}{2}}+2x^{-1}$

Allow the indices to be unprocessed and the coefficients to be unsimplified e.g. 2 of

$\left(f(x)=\right)10x+\frac{\frac{5}{2}x^{-\frac{3}{2}+1}}{-\frac{3}{2}+1}-\frac{2x^{-2+1}}{-2+1}$ The "f(x) =" is not required.

A1: $\left(f(x)=\right)10x-5x^{-\frac{1}{2}}+2x^{-1}+c$ oe seen on one line but condone the omission of + c

Allow the indices to be unprocessed and the coefficients to be unsimplified e.g. allow

$\left(f(x)=\right)10x+\frac{\frac{5}{2}x^{-\frac{3}{2}+1}}{-\frac{3}{2}+1}-\frac{2x^{-2+1}}{-2+1}$ The "f(x) =" is not required.

dM1: Uses $x=\frac{1}{4}$ and their value of y found from substituting their value of k and $x=\frac{1}{4}$ in $y=kx+2$ into their integrated function and finds the value of c.

Depends on the first method mark.

A1: $\left(f(x)=\right)10x-5x^{-\frac{1}{2}}+2x^{-1}+1$ or exact equivalent such as $\left(f(x)=\right)10x-\frac{5}{\sqrt{x}}+\frac{2}{x}+1$

The "f(x) =" is not required but the indices must now be processed and the coefficients simplified.

		(11 marks)

Question Number	Scheme	Marks
9(a)	$\frac{11\pi}{8}$	B1
		(1)
(b)	102	B1
	States $50\times2+2$ or other suitable reason	**d**B1
		(2)
(c)	$\left(\frac{3\pi}{8},-5\right)$	B1, B1
		(2)
		(5 marks)

Notes

Do <u>not</u> allow the use of degrees in this question.

If there is more than one answer for any part, score the final attempt.

Remember to check for answers written in the question.

(a)

B1: $\frac{11\pi}{8}$ or exact equivalent. Ignore any reference to the y coordinate, correct or incorrect but e.g. $\left(-1,\frac{11\pi}{8}\right)$ scores B0

(b)

B1: 102

dB1: Gives a suitable reason.

Allow a **correct** mathematical calculation. E.g. $50\times2+2$ or e.g. $50\times2+1+1$ or e.g. 100 + 2

If a **correct** mathematical calculation is seen do not be too concerned with the precise explanation for their calculation.

If they state 102 but the reason is **clearly incorrect** e.g. 3 roots between 0 and 2π and another 98 score B1B0

Allow an explanation '2 solutions every 2π radians plus another 2 in the next π radians'

Do not be concerned if it is ambiguous if the boundaries of x are included or not.

e.g. "there are 100 solutions for $0\leqslant x\leqslant 100\pi$ and then another 2 in the next π"

See the next page for a diagram of the scenario.

Depends on the first B mark

(c)

B1: One correct coordinate.

B1: $\left(\frac{3\pi}{8},-5\right)$

Note that $(\text{incorrect value}, -5)$ scores B1B0 not B0B1

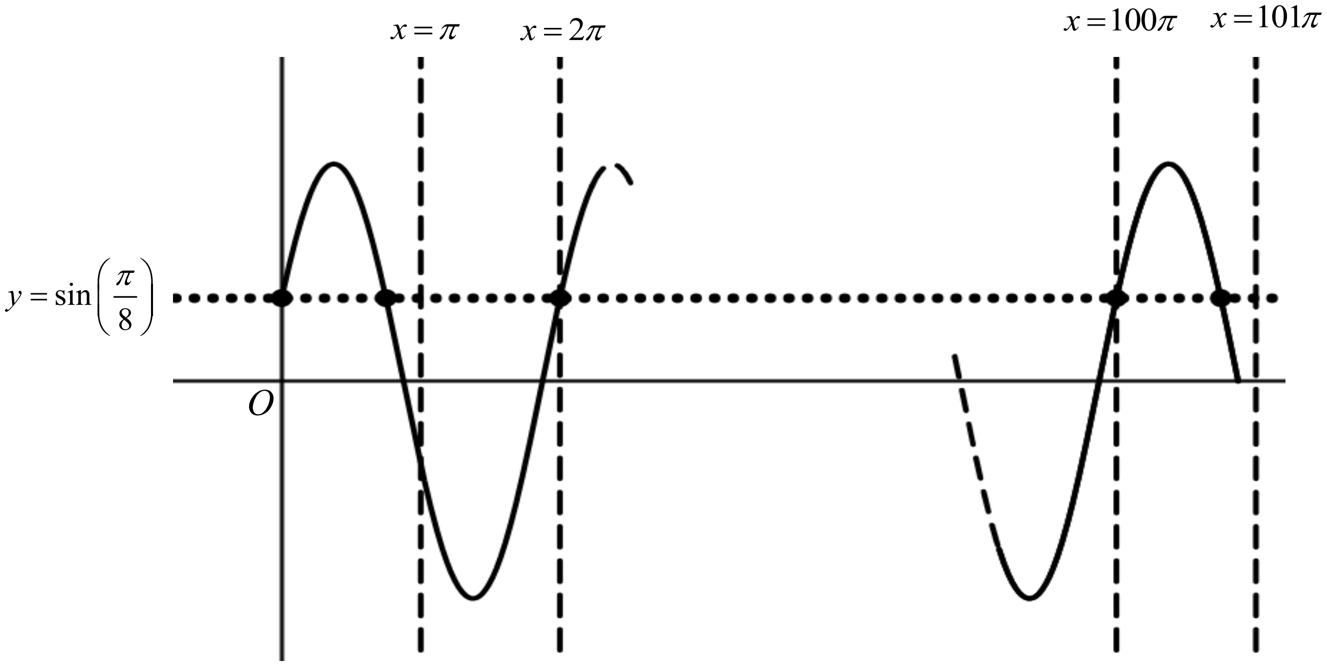
Allow answers to be written as coordinates or as $x=\ldots, y=\ldots$

Condone missing brackets e.g. $\frac{3\pi}{8},-5$ and condone punctuation other than a comma e.g. $\frac{3\pi}{8};-5$

The x coordinate must be written as a single fraction e.g. not $\frac{\pi}{2}-\frac{\pi}{8}$

The y coordinate must be written as single number so do not allow -5×1 for -5

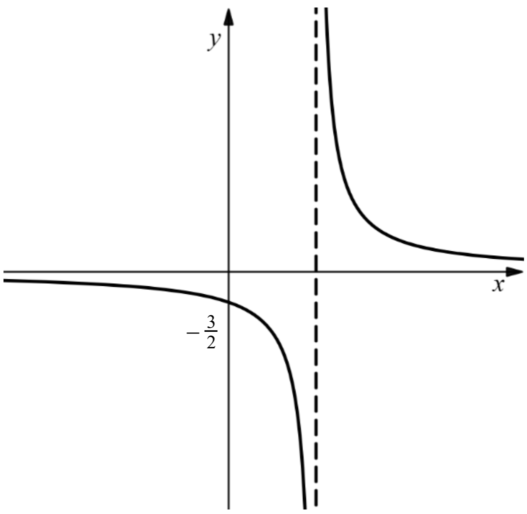
Do **not** allow coordinates the wrong way round e.g. $\left(-5,\frac{3\pi}{8}\right)$ scores B0B0



Note that the following were condoned as acceptable justification for 102 roots:

$0 \le x \le 100\pi$ = 100 solutions
π = 1 solutions   so 102
0 = 1 solution

b) $101 + 1 = 102$
- In 1π, they cut once
~~but~~ $2\pi \le x \le 100\pi$
~~at~~ ~~(100)~~ = 98 times
$0 \le x < 2\pi$, cut 3 times
and $100\pi \le x < 101\pi$, 1 more time
so $98 + 3 + 1 = 102$ times
$\Rightarrow$ 102 roots

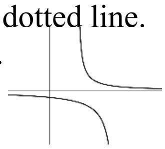
Question Number	Scheme		Marks
10(a)		Shape in quadrant one	M1
		Fully correct shape	A1
		Cuts the y-axis at $-\frac{3}{2}$	B1
			(3)

Notes

Marks cannot be scored without a sketch.

M1: Correct shape in quadrant 1 for the right hand branch. It must not cross either axis but be tolerant of functions that don't go down as far as the x-axis and do extend as far left as the y-axis.
Condone curves that touch the x or y-axis as long as they do not cross the axis.
Ignore any asymptotes, correct or incorrect, for this mark.
Ignore the other branch for this mark.

A1: Correct shape and position for whole curve. Be tolerant of slips of the pen but the intention should be clear. Look for the curve being in quadrants 1, 3 and 4 with a vertical asymptote in quadrant 1 and 4. The asymptote does **not** need to be drawn e.g. as a dotted line.
Do not allow this mark if the branches clearly overlap vertically. E.g.



The left hand branch must extend into quadrant 3 and should not intentionally dip away from the x-axis to e.g. create an intentional maximum.
There should be no horizontal asymptotes other than the x-axis.

B1: Cuts or touches the negative y-axis **once** at $-\frac{3}{2}$. Do **not** allow $-\frac{3k}{2k}$ but $\frac{3}{-2}$ is ok.
Must be in the correct position but allow as $-\frac{3}{2}$ or $y=-\frac{3}{2}$ or $\left(0,-\frac{3}{2}\right)$ and condone $\left(-\frac{3}{2},0\right)$ as long as it is in the correct place. Condone missing brackets e.g. $0,-\frac{3}{2}$
Allow to appear away from the sketch but it must be fully correct e.g. $\left(0,-\frac{3}{2}\right)$
If there is any ambiguity, the sketch takes precedence.

Note that there is no requirement in the question to label the asymptote so any labels, correct or incorrect can be ignored.

(b)	$\dfrac{3k}{x-2k}=12-x \Rightarrow 3k=12x-x^2-24k+2kx$	M1 A1
	$b^2-4ac=\left(12+2k\right)^2-4\times1\times27k$	**d**M1
	$\Rightarrow 144+48k+4k^2-108k=0 \Rightarrow k^2-15k+36=0$ *	A1*
		(4)

Notes

(b)

M1: Equates equations for curve with given line and attempts to form a quadratic equation in x. The terms do not need to be collected.

Look for $\dfrac{3k}{x-2k}=12-x$ (condoning slips), cross multiplies and proceeds to a quadratic equation which may be implied by an attempt at the discriminant.

Condone invisible brackets e.g. $\dfrac{3k}{x-2k}=12-x \rightarrow 3k=12-x\left(x-2k\right)$ as long as a quadratic equation in x is obtained and allow full recovery for the subsequent marks.

A1: Obtains a correct quadratic equation with brackets expanded which may be unsimplified e.g. $\dfrac{3k}{x-2k}=12-x \Rightarrow 3k=12x-x^2-24k+2kx$. May be implied.

dM1: Attempts b^2-4ac for their $ax^2+bx+c=0$ with both b and c expressions in terms of k. Condone slips/miscopies but they must be attempting the discriminant "correctly" e.g. if they expand to $3k=12x-x^2-24k+2kx$, they can't just find the discriminant of the rhs. May be seen embedded in an attempt at the quadratic formula.

A1*: Sets $b^2-4ac=0$ oe e.g. $b^2=4ac$, expands and simplifies to obtain $k^2-15k+36=0$ from **fully correct work**. Note that there must be at least one intermediate line between e.g. $\left(12+2k\right)^2-4\times1\times27k=0$ and $k^2-15k+36=0$

Note that the "= 0" must appear before the printed answer but may be as a preamble e.g. $b^2-4ac=0$ etc.

There must have been no incorrect statement within their solution e.g. $b^2-4ac<0$

<u>Note that is also possible to answer (b) in terms of *y* and this marks in the same way:</u>

$\dfrac{3k}{12-y-2k}=y \Rightarrow 3k=12y-y^2-2ky$ M1A1

$b^2-4ac=\left(2k-12\right)^2-4\times1\times3k$ **d**M1

$4k^2-48k+144-12k=0 \Rightarrow k^2-15k+36=0$ * A1

(c)	$k^2 - 15k + 36 = 0 \Rightarrow (k =) 3, 12$	B1
	$k = 3 \Rightarrow x^2 - 18x + 81 = 0\left(\text{or just } x = \frac{18}{2}\right) \Rightarrow x = 9 \Rightarrow y = 3$ or $k = 12 \Rightarrow x^2 - 36x + 324 = 0\left(\text{or just } x = \frac{36}{2}\right) \Rightarrow x = 18 \Rightarrow y = -6$	M1 A1
	$P = (18, -6)$	A1
		(4)

Notes

(c)

B1: Correct values for k. May be seen anywhere.
No method is required so just look for the correct values.
"$k =$" is not required and condone e.g. $x = 3, 12$

M1: Substitutes either of their values for k into their $x^2 - (12 + 2k)x + 27k = 0$ to set up an equation in x and solves to find a value for x **and** a corresponding value for y.
Note that as the quadratic in x has equal roots, the root may be found from $x = \frac{-b}{2a}$ so that the 3TQ in x may not be seen.
An alternative would be to solve C and l simultaneously with either of their values of k to find x or y via a 3TQ **and then find the value of the other variable**.

e.g $k = 3 \Rightarrow \frac{9}{x-6} = 12 - x \Rightarrow x^2 - 18x + 81 = 0 \Rightarrow x = ...(9) \Rightarrow y = ...(3)$

or e.g. $k = 3 \Rightarrow y = \frac{9}{12 - y - 6} \Rightarrow y^2 - 6y + 9 = 0 \Rightarrow y = ...(3) \Rightarrow x = ...(9)$

or

$k = 12 \Rightarrow \frac{36}{x-24} = 12 - x \Rightarrow x^2 - 36x + 324 = 0 \Rightarrow x = ...(18) \Rightarrow y = ...(-6)$

or e.g. $k = 12 \Rightarrow y = \frac{36}{12 - y - 24} = 12 - x \Rightarrow y^2 + 12y + 36 = 0 \Rightarrow y = ...(-6) \Rightarrow x = ...(18)$

In the above examples, the "= 0" in e.g. $x^2 - 18x + 81 = 0$ may be implied by their attempt to solve.
May be implied by correct or correct ft values of x and y.

A1: Correctly finds either $x = 9, y = 3$ **or** $x = 18, y = -6$

A1: Coordinates of P are $x = 18, y = -6$
Must clearly be identified as the only solution.
Allow answers to be written as coordinates or as $x = \ldots, y = \ldots$
Condone missing brackets e.g. $18, -6$ and condone punctuation other than a comma e.g. $18; -6$

		(11 marks)