Mark Scheme (Results)

Summer 2026

Pearson Edexcel International GCSE

In Further Pure Mathematics (4PM1)

Paper 01

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General Marking Guidance

All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.

Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.

Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.

There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.

All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.

Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.

When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.

Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

- **Types of mark**
    - M marks: method marks
    - A marks: accuracy marks – can only be awarded when relevant M marks have been gained
    - B marks: unconditional accuracy marks (independent of M marks)

- **Abbreviations**
    - cao – correct answer only
    - cso – correct solution only
    - ft – follow through
    - isw – ignore subsequent working
    - SC - special case
    - oe – or equivalent (and appropriate)
    - dep – dependent
    - indep – independent
    - awrt – answer which rounds to
    - eeoo – each error or omission

- **No working**
  If no working is shown then correct answers may score full marks
  If no working is shown then incorrect (even though nearly correct) answers score no marks.

- **With working**
  If it is clear from the working that the "correct" answer has been obtained from incorrect working, award 0 marks.

  If a candidate misreads a number from the question: eg. uses 252 instead of 255; follow through their working and deduct 2A marks from any gained provided the work has not been simplified. (Do not deduct any M marks gained.)

  If there is a choice of methods shown, then award the lowest mark, unless the subsequent working makes clear the method that has been used

  Examiners should send any instance of a suspected misread to review (but see above for simple misreads).

- **Ignoring subsequent work**
  It is appropriate to ignore subsequent work when the additional work does not change the answer in a way that is inappropriate for the question: eg. incorrect cancelling of a fraction that would otherwise be correct.

It is not appropriate to ignore subsequent work when the additional work essentially makes the answer incorrect eg algebra.

- **Parts of questions**
  Unless allowed by the mark scheme, the marks allocated to one part of the question CANNOT be awarded to another.

General Principles for Further Pure Mathematics Marking

(but note that specific mark schemes may sometimes override these general principles)

Method mark for solving a 3 term quadratic equation:

1. <u>Factorisation</u>:

$$\left(x^2+bx+c\right)=\left(x+p\right)\left(x+q\right), \text{ where } |pq|=|c| \text{ leading to } x=\ldots.$$

$$\left(ax^2+bx+c\right)=\left(mx+p\right)\left(nx+q\right) \text{ where } |pq|=|c| \text{ and } |mn|=|a| \text{ leading to } x=\ldots.$$

2. <u>Formula</u>:

Attempt to use the **correct** formula (shown explicitly or implied by working) with values for *a*, *b* and *c*, leading to $x=\ldots.$

3. <u>Completing the square:</u>

$$x^2+bx+c=0: \ \left(x\pm\tfrac{b}{2}\right)^2\pm q\pm c=0, \quad q\neq 0 \qquad \text{leading to } x=\ldots.$$

Method marks for differentiation and integration:

1. <u>Differentiation</u>

Power of at least one term decreased by 1. $\left(x^n\rightarrow x^{n-1}\right)$

2. <u>Integration:</u>

Power of at least one term increased by 1. $\left(x^n\rightarrow x^{n+1}\right)$

Use of a formula:

Generally, the method mark is gained by **either**

quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values

or, where the formula is <u>not</u> quoted, the method mark can be gained by implication from the substitution of <u>correct</u> values and then proceeding to a solution.

Answers without working:

The rubric states "Without sufficient working, correct answers <u>may</u> be awarded no marks".

General policy is that if it could be done "in your head" detailed working would not be required. (Mark schemes may override this eg in a case of "prove or show...."

Exact answers:

When a question demands an exact answer, all the working must also be exact. Once a candidate loses exactness by resorting to decimals the exactness cannot be regained.

Rounding answers (where accuracy is specified in the question)

Penalise only once per question for failing to round as instructed - ie giving more digits in the answers. Answers with fewer digits are automatically incorrect, but the is rule may allow the mark to be awarded before the final answer is given.

Question	Scheme	Marks
1 (a)	$\left(\mathrm{f}(x)=\right)-3\left(x^2-4x\right)-2$ or $-3\left(x^2-4x+\dfrac{2}{3}\right)$	M1
	$=-3\left[\left(x-2\right)^2-4\right]-2$ or $-3\left[\left(x-2\right)^2-4+\dfrac{2}{3}\right]$	M1
	$\left(\mathrm{f}(x)=\right)-3\left(x-2\right)^2+10$ or $a=-3,\quad b=-2,\quad c=10$	A1 [3]
(a)ALT	$a\left(x+b\right)^2+c=ax^2+2abx+ab^2+c$	M1
	$a=-3,\ 2ab=12,\ ab^2+c=-2\quad\Rightarrow a=\ldots,b=\ldots,c=\ldots$	M1
	$\left(\mathrm{f}(x)=\right)-3\left(x-2\right)^2+10$ or $a=-3,\quad b=-2,\quad c=10$	A1 [3]
(b)	$x=2$	B1ft [1]
		Total 4 marks

Part	Mark	Additional Guidance
(a)	M1	For factorising -3 out from the first two terms or all three terms of the given expression correctly, to achieve either: $-3\left(x^2-4x\right)-2$ or $-3\left(x^2-4x+\dfrac{2}{3}\right)$ oe
	M1	Attempts to complete the square, **regardless of any factor on the outside – follow through their factorisation.** ie $\left(x^2+px\right)+r$ or $\left(x^2+px+q\right)\Rightarrow\left(x\pm\dfrac{p}{2}\right)^2+k,\quad p\neq 0,\quad k\neq 0$
	A1	For $-3\left(x-2\right)^2+10$ or a, b and c correctly stated A correct answer without working can score M1M1A1
(a) ALT	M1	Multiplies out $a\left(x+b\right)^2+c=ax^2+2abx+ab^2+c$ correctly
	M1	Equates coefficients to obtains a value for a, b and c
	A1	For $-3\left(x-2\right)^2+10$ or a, b and c correctly stated
(b)	B1ft	For x = 2 **only** (Allow $x=\dfrac{4}{2}$), no other equations of lines seen Or follow through $x=-$ (their b) Note: Allow obtaining this via differentiation

Question number	Scheme	Marks
2	$-\frac{3}{2} < x < \frac{13}{2}$ $-7 \leqslant x \leqslant 5$ $-\frac{3}{2} < x \leqslant 5$	M1 A1 M1 A1 A1ft [5]
		Total 5 marks

Mark	**Additional Guidance** **Accept strict and non-strict inequalities for all M marks**
M1	For at least one correct value for their **inside** region e.g. $a < x < \frac{13}{2}$ or $-\frac{3}{2} < x < b$ or accept $x > -\frac{3}{2},\ x < b$ or accept $x > a,\ x < \frac{13}{2}$
A1	For $-\frac{3}{2} < x < \frac{13}{2}$, accept $x > -\frac{3}{2},\ x < \frac{13}{2}$ (allow "and ", but not "or " instead of the comma)
M1	For at least one correct value for their **inside** region e.g. $c \leqslant x \leqslant 5$ or $-7 \leqslant x \leqslant d$ or accept $x \geqslant c, x \leqslant 5$ or accept $x \geqslant -7,\ x \leqslant d$
A1	For $-7 \leqslant x \leqslant 5$, accept $x \geqslant -7, x \leqslant 5$ (allow "and ", but not "or " instead of the comma)
A1ft	For correct answer $-\frac{3}{2} < x \leqslant 5$, accept $x > -\frac{3}{2}$ **and** $x \leqslant 5$ oe e.g. $x \in (-\frac{3}{2}, 5]$ **or** follow through their previous two **inside** regions.

Question number	Scheme	Marks
3 (a)	$A=\frac{1}{2}r^2\left(\frac{\pi}{6}\right)-\frac{1}{2}r^2\sin\frac{\pi}{6}$	M1 M1
	$A=\frac{\pi}{12}r^2-\frac{1}{4}r^2=\frac{1}{12}r^2(\pi-3)$	A1*cso [3]
(b)	$\frac{1}{12}r^2(\pi-3)=6\Rightarrow r=...\quad\left(\sqrt{\frac{72}{\pi-3}}=22.54996581\right)$	M1
	$P="\sqrt{\frac{72}{\pi-3}}"+"\sqrt{\frac{72}{\pi-3}}"+"\sqrt{\frac{72}{\pi-3}}"\times\frac{\pi}{6}$	M1
	56.9	A1 [3]
		Total 6 marks

Part	Mark	Additional Guidance
(a)	M1	For correct use of the formula for area of a sector explicitly in any form
	M1	For correct use of the formula for area of a triangle explicitly in any form
	A1*cso	For a correct manipulation to simplify and then take out a factor to reach the **given result** without errors. Must have ***A* =** or **shaded area =** or **Area** = at some stage
(b)	M1	Sets the **correct** expression for the shaded area = 6 and reaches a value for r
	M1	Use of their r in a **correct** expression for the perimeter (arc length + two radii)
	A1	awrt 56.9 (ignore units)

Question number	Scheme	Marks
4 (a)	$(a=)4t-7=4\times3-7=5$	M1A1 [2]
(b)	$\left(s=\int\left(2t^2-7t+28\right)\mathrm{d}t\right)=\frac{2}{3}t^3-\frac{7}{2}t^2+28t(+C)$ $\frac{64}{3}=\frac{2}{3}\times2^3-\frac{7}{2}\times2^2+28\times2+C\Rightarrow C=...$ $(s=)\frac{2}{3}t^3-\frac{7}{2}t^2+28t-26$ $\frac{2}{3}\times6^3-\frac{7}{2}\times6^2+28\times6-26=...$ 160	M1 M1 A1 ddM1 A1 [5]
(c) (i)	$2\left(t-\frac{7}{4}\right)^2+\frac{175}{8}$ $\left(t-\frac{7}{4}\right)^2\geqslant0\Rightarrow v>0$ never at rest	M1A1 A1
ALT I	$\left(b^2-4ac=\right)(-7)^2-4(2)(28)=-175$ $-175<0$ no (real) solutions/roots Accept $-175<0$ never at rest	M1A1 A1
ALT II	$\frac{7\pm\sqrt{(-7)^2-4\times2\times28}}{2\times2}=\frac{7\pm5\sqrt{7}i}{4}$ or $=\frac{7\pm\sqrt{-175}}{4}$ Therefore no real solutions / imaginary roots/complex roots Or $\sqrt{-175}$ is impossible/no real root	M1A1 A1
(ii)	$\frac{175}{8}$	B1 [4]
		Total 11 marks

Part	Mark	Additional Guidance
(a)	M1	For a minimally acceptable attempt to differentiate v (see general guidance) **and** substitution of $t = 3$ into their expression.
	A1	For (a =) 5
(b)	M1	For a minimally acceptable attempt to integrate v, no power of any term to decrease. The + C does not need to be present for this mark.
	M1	Substitution of $t = 2$ and displacement $\frac{64}{3}$ into a changed expression, find $C = \ldots$
	A1	For the correct expression for displacement
	ddM1	Attempts to substitute $t = 6$ into their expression for displacement and finds a value of the displacement Dependant on both previous method marks
	A1	For 160
(c) (i)	M1	For attempting to complete the square of $v = a(t-b)^2 + c$, at least two values of a, b or c correct.
	A1	(**M1 in ePen**) For a correct completing the square of v
	A1	For a correct reason and conclusion, e.g. $(t-b)^2 \geqslant 0$, so $v > 0$ never at rest
ALT I (c) (i)	M1	For $(-7)^2 - 4(2)(28)$ **or** a correct discriminant
	A1	(**M1 in ePen**) For evaluating to a correct value -175
	A1	For stating their discriminant < 0 **and** no (real) solutions / never at rest
ALT II (c) (i)	M1	For use of a correct quadratic formula
	A1	(**M1 in ePen**) For evaluating to correct solutions
	A1	For stating there are no real solutions/imaginary roots/complex roots Or $\sqrt{-175}$ is impossible /no real roots
(ii)	B1	$\frac{175}{8}$ or 21.875 using any method, isw

Question number	Scheme	Marks
5 (a) (i)	$(\mathrm{f}(3)=)3^3-2\times3^2-9\times3+18$ $=0$	M1 A1cso [2]
(ii)	$[(x-3)](x^2+x-6)$ $[(x-3)](x+3)(x-2)$ $A(-3,\ 0)\quad B(2,\ 0)\quad C(3,\ 0)$	B1 B1 M1 A1A1cso [5]
(b)	$(\mathrm{f}'(x)=)3x^2-4x-9=0$ $D\,/\,\text{Max}:(x=)\dfrac{2-\sqrt{31}}{3}\quad E\,/\,\text{Min}:(x=)\dfrac{2+\sqrt{31}}{3}$	M1 A1 A1cso [3]
(c)	$\left(\int\left(x^3-2x^2-9x+18\right)\mathrm{d}x\right)=\dfrac{x^4}{4}-\dfrac{2x^3}{3}-\dfrac{9x^2}{2}+18x$ $\left[\dfrac{x^4}{4}-\dfrac{2x^3}{3}-\dfrac{9x^2}{2}+18x\right]_0^{"2"}$ $=\dfrac{"2"^4}{4}-\dfrac{2\times"2"^3}{3}-\dfrac{9\times"2"^2}{2}+18\times"2"(-0)$ $\left(=\dfrac{50}{3}\right)$ $\left[\dfrac{x^4}{4}-\dfrac{2x^3}{3}-\dfrac{9x^2}{2}+18x\right]_{"2"}^{3}$ $=\left(\dfrac{3^4}{4}-\dfrac{2\times3^3}{3}-\dfrac{9\times3^2}{2}+18\times3\right)-\left(\dfrac{"2"^4}{4}-\dfrac{2\times"2"^3}{3}-\dfrac{9\times"2"^2}{2}+18\times"2"\right)$ $\left(=\dfrac{63}{4}-\dfrac{50}{3}\right)=\left(-\dfrac{11}{12}\right)$ $=\ "\dfrac{50}{3}"+\left\|"-\dfrac{11}{12}"\right\|$ or $"\dfrac{50}{3}"-"-\dfrac{11}{12}"$ $\dfrac{211}{12}$	M1 M1 M1 dddM1 A1cso [5]
		Total 15 marks

Part	Mark	Additional Guidance
(a) (i)	M1	Attempts to substitute $x = \pm 3$ into f(x)
	A1cso	For a correct substitution with x = 3 and obtains 0, no errors.
(ii)	B1	For finding the 3TQ factor in the form $x^2 + x \pm c$ or $x^2 \pm bx - 6$
	B1	For finding the correct 3TQ factor $x^2 + x - 6$
	M1	For a correct factorisation of their 3TQ or any non-calculator method to solve their 3TQ equation See general guidance.
	A1	For any two coordinates or two correct x values provided y = 0 seen or implied
	A1cso	For $A(-3, 0)$ $B(2, 0)$ $C(3, 0)$ given as correct coordinates only
(b)	M1	For a minimally acceptable attempt (see general guidance) to differentiate f (x)
	A1	For the correct derivative and sets it equal to 0, but it can be implied by later solving
	A1cso	For correct **exact** x values for the corresponding points Must be clear which exact x value is for D/max and which exact x value is for E/min Allow solving from a calculator
(c)	M1	Attempts to integrate f(x) - see general guidance, no power of x to decrease
	M1	For substitution of limits of x = "their 2" (must come from part (a)) and x = 0 (0 doesn't need to be shown, but 0 needs to be the lower limit) If no explicit substitution seen, the substitution of 2 can only be implied by $\frac{50}{3}$ oe from a correct integral
	M1	For substitution of limits of x = 3 and x = "their 2" (must come from part (a)) and subtract either way round If no explicit substitution seen, the substitution of 2 can only be implied by $\frac{50}{3}$ oe from a correct integral If no explicit substitution seen, the substitution of 3 can only be implied by $\frac{63}{4}$ oe from a correct integral
	dddM1	For the correct method of finding the required area e.g. Taking the modulus of the second integral and adding to the first, oe Dependent on all previous method marks
	A1cso	For the correct answer

<table>
<tr><th>Question number</th><th>Scheme</th><th>Marks</th></tr>
<tr><td>6 (a)</td><td>

x	-1	-0.5	0	0.5	1
y	**5.75**	1.96	1.25	**1.96**	**5.75**

</td><td>B1
B1

[2]</td></tr>
<tr><td>(b)</td><td>All five points plotted.

Smooth curve joins all five points</td><td>B1ft

B1ft
[2]</td></tr>
<tr><td>(c)</td><td>$\left(x^2+2=\log_3(12x+12)\right)\Rightarrow 3^{(x^2+2)}=12x+12$
$\left(\frac{1}{4}\times 3^{(x^2+2)}=3x+3\right)\Rightarrow \frac{3^{(x^2+2)}}{4}-1=3x+2$

Draws the line $y=3x+2$
$(x=)-0.2,\ 0.9$</td><td>M1
M1
M1
A1
[4]</td></tr>
<tr><td>(c)ALT</td><td>$\frac{3^{(x^2+2)}}{4}-1=mx+c\Rightarrow 3^{(x^2+2)}=4mx+4c+4$
$x^2=\log_3\left(4mx+4c+4\right)-2\Rightarrow m=...,\ c=...$

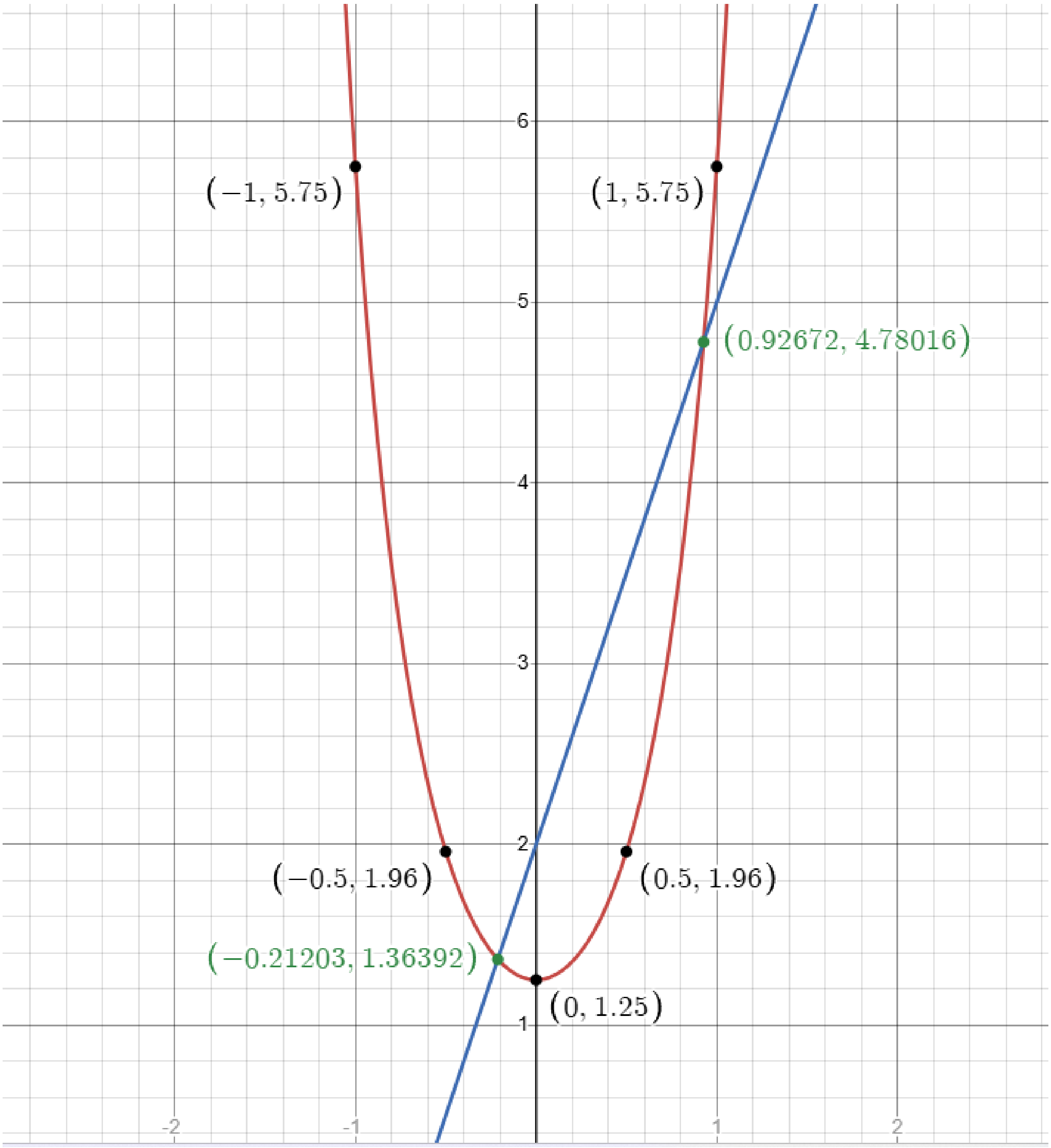
Draws the line $y=3x+2$
$(x=)-0.2,\ 0.9$</td><td>M1

M1

M1
A1
[4]</td></tr>
<tr><td colspan="3" align="right">Total 8 marks</td></tr>
</table>

Part	Mark	Additional Guidance
(a)	B1	For at least 1 value correct to 2dp
	B1	For all 3 values correct to 2dp
(b)	B1ft	For all five points plotted within half a square ft their values from the table.
	B1ft	For all five points joined by a smooth curve ft their points plotted.
(c)	M1	Correctly removes log, and obtains $3^{(x^2+2)}$ = minimum two terms linear expression of x
	M1	Rearranges to the form $\frac{3^{(x^2+2)}}{4}-1=$ minimum two terms linear expression of x
	M1	Correctly draws their line in the form of $y=3x+k,\quad k\neq 0$ oe, within half a square tolerance.
	A1	For **both** correct roots. Accept $-0.3/-0.2/-0.1,\quad 0.8/0.9/1.0/1$
(c) ALT	M1	Sets the given y equal to $m\,x+c$ and rearranges to $3^{(x^2+2)}=\ldots$
	M1	Correctly takes log base 3 to both sides, makes x^2 the subject, then compares to find a value for m and c
	M1	Correctly draws their line in the form of $y=3x+k,\quad k\neq 0$, within half a square tolerance.
	A1	For **both** correct roots. Accept $-0.3/-0.2/-0.1,\quad 0.8/0.9/1.0/1$

Useful Sketch:



Question number	Scheme	Marks
7 (a)	$\sqrt[3]{\frac{27}{125}}\left(=\frac{3}{5}\right)$	B1
	$125\times\left("\frac{3}{5}"\right)^2\ (=45)$	M1
	$125+(9-1)d=125\times\left("\frac{3}{5}"\right)^2 \Rightarrow d=...$	M1
	$(d=)-10$	A1
	$(S_{20}=)\frac{20}{2}(2\times125+(20-1)\times"-10")$	ddM1
	$=600$	A1 [6]
(b)	$(S_\infty=)\frac{125}{1-"\frac{3}{5}"}$	M1
	312.5	A1 [2]
(c)	$\frac{125\left(1-\left("\frac{3}{5}"\right)^n\right)}{1-"\frac{3}{5}"}>300\left(\Rightarrow 1-\left("\frac{3}{5}"\right)^n>\frac{24}{25}\right)$ oe	M1
	$\Rightarrow\left("\frac{3}{5}"\right)^n<\frac{1}{25}$	dM1
	$n\log\left("\frac{3}{5}"\right)<\log\left(\frac{1}{25}\right)$ oe e.g. $n>\log_{\frac{3}{5}}\left(\frac{1}{25}\right)$	ddM1
	$\Rightarrow n>\frac{\log\left(\frac{1}{25}\right)}{\log\left("\frac{3}{5}"\right)}\Rightarrow n>6.301$ oe $\Rightarrow (n=)7$	dddM1A1 [5]
		Total 13 marks

Part	Mark	Additional Guidance
(a)	B1	For $\sqrt[3]{\frac{27}{125}}$ or $\frac{3}{5}$,ignoring labelling
	M1	For a correct third term of *G,* follows through their *r*
	M1	For a correct method to find the common difference, follows through their *r*
	A1	For $(d=)-10$
	ddM1	For a correct substitution into the sum of the arithmetic series using their *d* and the given first term Dependent on both previous method marks.
	A1	For 600
(b)	M1	For correct substitution into the formula for sum to infinity, follows through their $\lvert r \rvert < 1$
	A1	For 312.5 or $\frac{625}{2}$ (must come from *r* found correctly)
In part c, allow working in inequality ($<, \leqslant, >, \geqslant$) or equation (=) for all Method marks		
(c)	M1	Uses the correct formula for the sum of a geometric series to set up an **inequality or equation** in terms of *n*, follows through their *r* , must be using $a = 125$
	dM1	For simplifying (allow slips in simplification) their **inequality or equation** in *n* to the form $(\text{"their } r\text{"})^n \text{ "<" } d \quad d \neq 0$ **or** $p(\text{"their } r\text{"})^n \text{ "<" } q \quad p,q \neq 0$ Dependent on the previous method mark.
	ddM1	Takes logarithms (any base) of their **exponential equation or inequality <u>correctly</u>** on both sides and correctly uses the power law to reach $n\log(a) < \log(b)$. oe or 'undo-log' their **exponential equation or inequality <u>correctly</u>** to get $\log_c d < e$ This isn't a dependent mark, but the candidate must be able to correctly take logs of both sides and use the power law or correctly undo log their equation/inequality. *r* can be any value carried through from part (a) for this mark. They cannot be awarded this mark if they are taking the log of a negative number. The inequality doesn't need to have been reversed at this point. Dependent on the previous two method marks.
	dddM1	For finding a value for *n* Dependent on all previous method marks.
	A1	For $(n=)$ 7 If work in inequalities, all inequalities must be correct throughout. If work in equations, they must justify why *n* is equal to 7. E.g. when $n = 6$ $S_6 = 297.92$, when $n = 7$, $S_7 = 303.752$, so $n = 7$

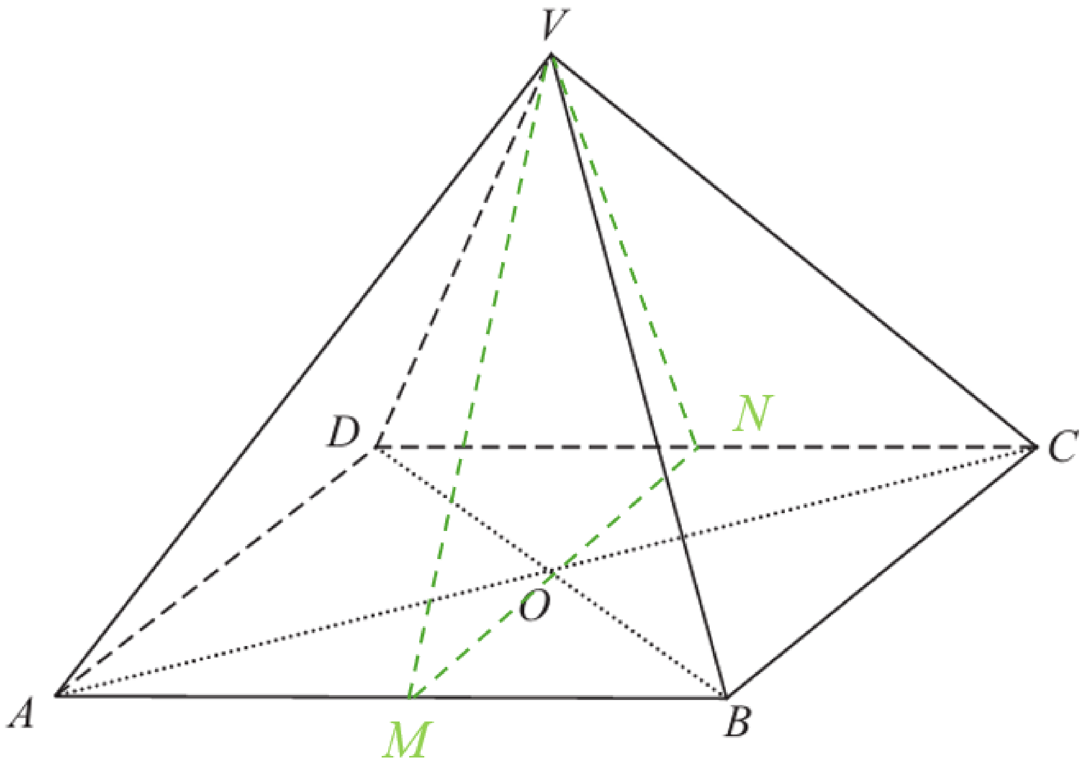
Question number	Scheme Allow x is not present or substituted by any positive value in part a and part b	Marks
8 (a)	$(AC =)\sqrt{(2x)^2+(2x)^2}\ \left(=\sqrt{8}x\right)$ **or** $(AO =)\sqrt{(x)^2+(x)^2}\ \left(=\sqrt{2}x\right)$	M1
	$\left(\sqrt{8}x\right)^2=(2x)^2+(2x)^2-2\times 2x\times 2x\cos\angle AVC$ **or** $\sin\left(\frac{1}{2}\angle AVC\right)=\frac{\sqrt{2}x}{2x}$ oe **or** $\cos\left(\frac{1}{2}\angle AVC\right)=\frac{\sqrt{2}x}{2x}$ oe with $VO=\sqrt{2}x$ (stated or implied) **or** $\tan\left(\frac{1}{2}\angle AVC\right)=\frac{\sqrt{2}x}{\sqrt{2}x}$ oe with $VO=\sqrt{2}x$ (stated or implied)	M1
	$(\angle AVC =)\cos^{-1}\left(\frac{(2x)^2+(2x)^2-\left(\sqrt{8}x\right)^2}{2\times 2x\times 2x}\right)$ **or** $(\angle AVC =)2\times\sin^{-1}\left(\frac{\sqrt{2}x}{2x}\right)$ **or** $(\angle AVC =)2\times\cos^{-1}\left(\frac{\sqrt{2}x}{2x}\right)$ **or** $(\angle AVC =)2\times\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{2}x}\right)$ **or** $AO=VO$ therefore triangle AVC is isosceles and $AVC = 2\times 45°$	M1
	90°	A1*cso [4]
(a) ALT	Using Pythagoras:(in triangle ABC) $AC=\sqrt{(2x)^2+(2x)^2}$ or $AC^2=(2x)^2+(2x)^2$	M1
	A **clear indication** that they are using triangle ABC or ADC or the square base to find the above AC, shown in any stage of their working or a diagram/sketch	M1
	Using triangle VAC: $\sqrt{(2x)^2+(2x)^2}=AC$ or $(2x)^2+(2x)^2=AC^2$ Allow assuming VAC is a right-angle triangle	M1
	State it fits Pythagoras theorem, angle $AVC=90°$ without error	A1

(b)	Let M be the midpoint of AB and let N be the midpoint of DC $(VM = VN =)\sqrt{(2x)^2 - (x)^2} = (\sqrt{3}x)$ **or** $VO = \sqrt{2}x$ with $MO / NO = x$	M1
	$(\angle MVN =) = \cos^{-1}\left(\dfrac{(''\sqrt{3}x'')^2 + (''\sqrt{3}x'')^2 - (2x)^2}{2\times(''\sqrt{3}x'')\times(''\sqrt{3}x'')}\right)$ **or** $(\angle MVN =) 2\times\sin^{-1}\left(\dfrac{x}{\sqrt{3}x}\right)$ **or** $(\angle MVN =) 2\times\cos^{-1}\left(\dfrac{\sqrt{2}x}{\sqrt{3}x}\right)$ **or** $(\angle MVN =) 2\times\tan^{-1}\left(\dfrac{x}{\sqrt{2}x}\right)$	M1
	$\angle MVN = 70.5^{\circ}$	A1 [3]
(c)	$\frac{1}{2}\times 2x\times 2x\times\sin 60 = 49\sqrt{3}$ oe e.g. $\frac{1}{2}\times 2x\times\sqrt{3}x = 49\sqrt{3}$	M1
	$x = \ldots$	dM1
	$(x =) 7$	A1 [3]
		Total 10 marks

Part	Mark	Additional Guidance
(a)	M1	For correct use of Pythagoras theorem or any correct appropriate trigonometry to find the length AC or AC^2 or AO/CO or AO^2 / CO^2 , simplified or unsimplified length
	M1	For a correct use of cosine rule for angle AVC **Or** uses of any correct trigonometry to form a correct equation with ½ angle AVC, along with a correct length of VO if needed in their equation.
	M1	For any correct and complete method to find angle AVC. $(\angle AVC =)$
	A1*cso	Correct angle found without errors
ALT using Pythagoras please see main scheme		
(b)	M1	For correct Pythagoras theorem to find the lengths of VM or VN or VO, simplified or unsimplified length
	M1	For any complete and correct trigonometry to find angle MVN
	A1	For awrt 70.5°
(c)	M1	For any correct equation for the area of the triangle ABV formed e.g. $\frac{1}{2}\times 2x\times 2x\times\sin 60^{\circ} = 49\sqrt{3}$ or $\frac{1}{2}\times 2x\times$ their $VM = 49\sqrt{3}$,their VM must be found from a correct method
	dM1	Correct method of solving their equation (usual rule) (allow use of a calculator, but their answer must be correct for their equation) and finds a **positive** value of x Dependent on the previous method mark.
	A1	$(x =) 7$ only

Solutions based on congruency, please send to Review.

Useful Sketch:



Question number	Scheme	Marks
9	$\log_{81} x + \log_9 x + \log_3 x^2 = 22$	
	$\dfrac{\log_3 x}{\log_3 81} + \dfrac{\log_3 x}{\log_3 9} + \log_3 x^2 (= 22)$	M1
	$\dfrac{\log_3 x}{4} + \dfrac{\log_3 x}{2} + \log_3 x^2 (= 22)$ oe	dM1
	$\dfrac{11}{4}\log_3 x (= 22)$ oe	ddM1
	$\log_3 x = 8$	A1
	$x = 3^8$	M1
	$x = 6561$ (accept 3^8)	A1 [6]
ALT I	$\log_{81} x + \log_9 x + \log_3 x^2 = 22$	
	$\dfrac{\log_9 x}{\log_9 81} + \log_9 x + \dfrac{\log_9 x^2}{\log_9 3} (= 22)$	M1
	$\dfrac{\log_9 x}{2} + \log_9 x + 2\log_9 x^2 (= 22)$ oe	dM1
	$\dfrac{11}{2}\log_9 x (= 22)$ oe	ddM1
	$\log_9 x = 4$	A1
	$x = 9^4$	M1
	$x = 6561$ (accept 9^4)	A1 [6]
ALT II	$\log_{81} x + \log_9 x + \log_3 x^2 = 22$	
	$\log_{81} x + \dfrac{\log_{81} x}{\log_{81} 9} + \dfrac{\log_{81} x^2}{\log_{81} 3} (= 22)$	M1
	$\log_{81} x + 2\log_{81} x + 4\log_{81} x^2 (= 22)$ oe	dM1
	$11\log_{81} x (= 22)$ oe	ddM1
	$\log_{81} x = \dfrac{22}{11}$	A1
	$x = 81^2$	M1
	$x = 6561$ (accept 81^2)	A1 [6]
		Total 6 marks

Part	Mark	Additional Guidance
	M1	At least one correct change of base of log.
	dM1	For correctly changing all terms to the same base, and applies $\log_3 81 = 4$ and $\log_3 9 = 2$ Dependent on the previous M mark
	ddM1	For applies **correct** log rules to combine **their** lhs to a single log term **correctly**, ignore rhs Dependent on both previous M marks
	A1	For a correct equation with a single log term with no coefficient in front of the log: $\log_3 x = 8$, accept $\log_3 x^{\frac{11}{4}} = 22$, can be implied by the correct final answer.
	M1	For correctly removes log, if method not seen, it can only be implied by a correct final answer Note: This is an independent method mark
	A1	For 6561 or 3^8 isw
ALT I	M1	At least one fully correct change of base of log.
	dM1	For correctly changing all terms to the same base, and applies $\log_9 81 = 2$ and $\log_9 3 = \frac{1}{2}$ Dependent on the previous M mark
	ddM1	For applies **correct** log rules to combine **their** lhs to a single log term **correctly**, ignore rhs Dependent on both previous M marks
	A1	For a correct equation with a single log term with no coefficient in front of the log: $\log_9 x = 4$, accept $\log_9 x^{\frac{11}{2}} = 22$, can be implied by the correct final answer
	M1	For correctly removes log, if method not seen, it can only be implied by a correct final answer Note: This is an independent method mark
	A1	For 6561 or 9^4 isw
ALT II	M1	At least one fully correct change of base of log.
	dM1	For correctly changing all terms to the same base, and applies $\log_{81} 9 = \frac{1}{2}$ and $\log_{81} 3 = \frac{1}{4}$ Dependent on the previous M mark
	ddM1	For applies **correct** log rules to combine **their** lhs to a single log term **correctly**, ignore rhs Dependent on both previous M marks
	A1	For a correct equation with a single log term with no coefficient in front of the log: $\log_{81} x = 2$, accept $\log_{81} x^{11} = 22$, can be implied by the correct final answer
	M1	For correctly removes log, if method not seen, it can only be implied by a correct final answer Note: This is an independent method mark
	A1	For 6561 or 81^2 isw

If change to other base seen e.g. $\log x$ **or** $\log_{10} x$ **or** $\log_x 3$ **, mark using the same principle, if not sure how to mark, please sent to Review**

Question number	Scheme	Marks
10	$x-2=\sqrt{x+4}$ or $y+2=y^2-4$	M1
	$x^2-5x=0$ or $y^2-y-6=0$	M1
	$x(x-5)=0\Rightarrow x=5$ $(y-3)(y+2)=0$	M1
	$\Rightarrow y=3$ $\Rightarrow y=3$	A1
	(At point A) $y=2$	B1
	$\pi\int_0^3 (y+2)^2\ \mathrm{d}y-\pi\int_2^3 (y^2-4)^2\ \mathrm{d}y$ oe	dddM1
	e.g. $\pi\left(\int_0^3 (y^2+4y+4)\ \mathrm{d}y-\int_2^3 (y^4-8y^2+16)\ \mathrm{d}y\right)$	
	$=(\pi)\left(\left[\frac{y^3}{3}+\frac{4}{2}y^2+4y\right]_{("0")}^{("3")}-\left[\frac{y^5}{5}-\frac{8}{3}y^3+16y\right]_{("2")}^{("3")}\right)$	M1
	$=(\pi)\left\{\left(\frac{"3"^3}{3}+\frac{4}{2}\times"3"^2+4\times"3"\right)-\left(\left(\frac{"3"^5}{5}-\frac{8}{3}\times"3"^3+16\times"3"\right)-\left(\frac{"2"^5}{5}-\frac{8}{3}\times"2"^3+16\times"2"\right)\right)\right\}$	dM1
	$=\left(39\pi-\frac{113}{15}\pi\right)=\frac{472}{15}\pi$	A1cso
		Total 9 marks

Part	Mark	Additional Guidance
	M1	For correctly equating the equations of l and C.
	M1	For correctly expanding and simplifying to a 2TQ or 3TQ
	M1	For a correct method to solve their quadratic equation. For a 2TQ, the non-zero solution must be **correct** for their equation, no need to see $x = 0$. For a 3TQ, this must be a minimally acceptable attempt (see General Guidance). If use of a calculator, their solution must be correct for their equations
	A1	For $y = 3$ or (5,3) seen in the working or graph
	B1	For $y = 2$ or (0,2) seen in the working or graph or 2 labelled on the y axis
	dddM1	For a **correct** statement for the required volume, including **correct** limits and π Condone dy is missing or using dx If two volumes are done separately, this mark can be implied by later work. If subtract the wrong way round, this mark can be recovered by later correctly reverse back Dependent all previous M marks
	M1	For a minimally acceptable attempt to integrate either of their expressions in y (see General Guidance) – there must be at least 3 terms to integrate. π and limits do not need to be seen
	dM1	For correctly substituting their y limits **only** to either **integrated** expression in y and subtracts either way round, each nonzero limit must be substituted explicitly and correctly at least once. (A substitution of 0 need not be seen). If no explicit substitution shown, this mark can only be implied by correctly integrated **expressions** with a correct **final** answer. π does not need to be seen, dependent on the previous M mark only
	A1cso	For $\frac{472}{15}\pi$

Question number	Scheme	Marks
11 (a)	$(\alpha+\beta)^3 = \alpha^3 + 3\alpha^2\beta + 3\alpha\beta^2 + \beta^3$	B1 [1]
(b)	$\alpha+\beta=2$ **and** $\alpha\beta=5$	B1
	$\alpha^3+\beta^3 = (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)$ oe	M1
	$=("2")^3 - 3("5")("2")$	
	-22	A1 [3]
(c)	$\alpha^5+\beta^5 = (\alpha+\beta)^5 - 5\alpha^4\beta - 5\alpha\beta^4 - 10\alpha^3\beta^2 - 10\alpha^2\beta^3$	
	$=(\alpha+\beta)^5 - 5\alpha\beta(\alpha^3+\beta^3) - 10(\alpha\beta)^2(\alpha+\beta)$ oe	M1
	$=2^5 - 5\times"5"\times"-22" - 10\times"5^2"\times"2"$	dM1
	$=82$ *	A1*cso [3]
(d)	$\left(\alpha^3+\frac{1}{\alpha^2}+\beta^3+\frac{1}{\beta^2}\right) = \alpha^3+\beta^3+\frac{(\alpha+\beta)^2-2\alpha\beta}{(\alpha\beta)^2}$ oe	M1
	$="-22"+\frac{("2")^2-2\times"5"}{("5")^2}\left(=-\frac{556}{25}\right)$	M1
	$\left(\left(\alpha^3+\frac{1}{\alpha^2}\right)\left(\beta^3+\frac{1}{\beta^2}\right)=\alpha^3\beta^3+\frac{\alpha^3}{\beta^2}+\frac{\beta^3}{\alpha^2}+\frac{1}{\alpha^2\beta^2}=\right)(\alpha\beta)^3+\frac{\alpha^5+\beta^5}{(\alpha\beta)^2}+\frac{1}{(\alpha\beta)^2}$ oe	M1
	$=("5")^3+\frac{82}{("5")^2}+\frac{1}{("5")^2}\left(=\frac{3208}{25}\right)$	M1
	$x^2-\left("-\frac{556}{25}"\right)x+"\frac{3208}{25}"(=0)$ oe	M1
	$25x^2+556x+3208=0$	A1 [6]
		Total 13 marks

Part	Mark	Additional Guidance
(a)	B1	$\alpha^3+3\alpha^2\beta+3\alpha\beta^2+\beta^3$, accept $\alpha^3+3\left(\alpha^2\beta+\alpha\beta^2\right)+\beta^3$ or $\alpha^3+3\alpha\beta(\alpha+\beta)+\beta^3$
(b)	B1	For the correct values of $\alpha+\beta=2$ **and** $\alpha\beta=5$
	M1	For substituting their values of $\alpha+\beta$ and $\alpha\beta$ to a rearranged expression in the form of $(\alpha+\beta)^3\pm k\alpha\beta(\alpha+\beta)$ oe, $k\neq 0$, ready for substitution of values of $\alpha+\beta$ and $\alpha\beta$
	A1	For -22
(c)	M1	Attempts to rearrange to obtain $\alpha^5+\beta^5=(\alpha+\beta)^5\pm 5\alpha\beta\left(\alpha^3+\beta^3\right)\pm 10(\alpha\beta)^2(\alpha+\beta)$ oe and ready for substitution of values of $\alpha^3+\beta^3,\ \alpha+\beta,\ \alpha\beta$
	dM1	For the substitution of their values to a correct rearrangement Dependent on previous mark.
	A1*cso	Obtains the correct answer with no errors
(d)	M1	Correct algebra for sum: $\alpha^3+\beta^3+\dfrac{(\alpha+\beta)^2-2\alpha\beta}{(\alpha\beta)^2}$ oe, ready for substitution of values
	M1	Correct substitution of their values of $\alpha^3+\beta^3,\ \alpha+\beta,\ \alpha\beta$ into a correct algebra for the sum, can be implied by the correct sum found
	M1	Correct algebra for product: $(\alpha\beta)^3+\dfrac{\alpha^5+\beta^5}{(\alpha\beta)^2}+\dfrac{1}{(\alpha\beta)^2}$ oe, ready for substitution of values
	M1	Correct substitution of their values of $\alpha+\beta,\ \alpha\beta$ and 82 for $\alpha^5+\beta^5$, into a correct algebra for the product, can be implied by the correct product found
	M1	For $x^2-\left("-\dfrac{556}{25}"\right)x+"\dfrac{3208}{25}"$ oe, using **their** new product and sum of roots.
	A1	For the **equation** shown. Allow any equivalent integer coefficients.

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