Cambridge International A Level

MATHEMATICS **9709/32**

Paper 3 Pure Mathematics 3 **May/June 2026**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme contains the instructions and marking guidance that our examiners used to mark candidate responses for this component.

Before they start marking, examiners complete a standardisation process. They review a range of candidate responses and discuss the mark scheme in detail to agree which answers can and cannot be accepted. Examiners award marks where it is clear that candidate responses have met the requirements of the mark scheme. These requirements may vary between different components or different versions of the same component, depending on the exact requirements of the question and the candidate responses seen during the standardisation process.

Sometimes, our mark schemes may allow marks to be awarded to responses which contain elements that are factually incorrect or for content not covered by the syllabus. We do this in response to the demands of a specific question to support candidates and make sure that our assessments are fair. However, these allowances should not be used as a guide for teaching and learning.

We cannot discuss the mark scheme with schools, and we recommend that schools read the mark scheme together with the assessment material and the Principal Examiner Report for Teachers.

This document consists of **26** printed pages.

General marking principles

All examiners must apply these marking principles when marking candidate responses.

1	Examiners must award marks for each response in line with: • the specific **content** defined in the mark scheme • the specific **skills** defined in the mark scheme or in the level descriptions • the **standard of response** required as exemplified by the standardisation scripts.
2	Examiners must award **whole marks** (not half marks, or other fractions).
3	Examiners must award marks **positively**. This means that: • marks are not deducted for errors or omissions • marks are awarded for correct/valid answers as defined in the mark scheme and also for **other valid answers** which go beyond the scope of the syllabus and mark scheme referring to your team leader as appropriate • the quality of spelling, punctuation and grammar are judged **only** when the mark scheme indicates that these features are **specifically assessed** in the question. However, the meaning of all responses must be unambiguous.
4	Examiners must **consistently** apply the requirements of the mark scheme and any rules for what to do when candidates have not followed instructions correctly.
5	Examiners must use the **full range of marks** defined in the mark scheme, unless limited by the quality of the candidate responses seen.
6	Examiners must award marks according to the mark scheme and **must not** award marks with grade thresholds or grade descriptions in mind.

Mathematics-Specific Marking Principles

1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.

2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.

3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.

4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).

5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.

6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Annotations guidance for centres

Examiners use a system of annotations as a shorthand for communicating their marking decisions to one another. Examiners are trained during the standardisation process on how and when to use annotations. The purpose of annotations is to inform the standardisation and monitoring processes and guide the supervising examiners when they are checking the work of examiners within their team. The meaning of annotations and how they are used is specific to each component and is understood by all examiners who mark the component.

We publish annotations in our mark schemes to help centres understand the annotations they may see on copies of scripts. Note that there may not be a direct correlation between the number of annotations on a script and the mark awarded. Similarly, the use of an annotation may not be an indication of the quality of the response.

The annotations listed below were available to examiners marking this component in this series.

Annotations

Annotation	Meaning
^	More information required
A0	Accuracy mark awarded zero
A1	Accuracy mark awarded one
B0	Independent accuracy mark awarded zero
B1	Independent accuracy mark awarded one
B2	Independent accuracy mark awarded two
BOD	Benefit of the doubt
BP	Blank Page
✗	Incorrect
Dep	Used to indicate DM0 or DM1

Annotation	Meaning
DM1	Dependent on the previous M1 mark(s)
FT	Follow through
	Indicate working that is right or wrong
Highlighter	Highlight a key point in the working
ISW	Ignore subsequent work
J	Judgement
JU	Judgement
M0	Method mark awarded zero
M1	Method mark awarded one
M2	Method mark awarded two
MR	Misread
O	Omission or Other solution
Off-page comment	Allows comments to be entered at the bottom of the RM marking window and then displayed when the associated question item is navigated to.
On-page comment	Allows comments to be entered in speech bubbles on the candidate response.
PE	Judgment made by the PE
Pre	Premature approximation
SC	Special case
SEEN	Indicates that work/page has been seen

Annotation	Meaning
SF	Error in number of significant figures
✓	Correct
TE	Transcription error
XP	Correct answer from incorrect working

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	Commence division and reach partial quotient of the form $2x^2 \pm kx$ or $4x^3 - 4x^2 + 7x + 2 = (2x - 1)(Ax^2 + Bx + C) + D$ with $A = 2$ and $B = \pm k$	**M1**	$k \neq 0$
	Obtain quotient $2x^2 - x + 3$ Or: state the factor as above and then at least $A = 2$ and $B = -1$, $C = 3$	**A1**	Allow the A marks when seen and ISW, unless they swap quotient and remainder in their conclusion, in which case A1A0. Do not need to state which is which.
	Obtain remainder 5 Or: $D = 5$	**A1**	Allow $2x^2 - x + 3 + \frac{5}{2x-1}$, but saying 'the remainder is $\frac{5}{2x-1}$' is A0. **SC** Allow three marks for correct answer with no working **SC** B1 if they use $\mathrm{f}\left(\frac{1}{2}\right) = 5$ to obtain the remainder.
			$\begin{array}{r\|rrrr} & & 2x^2 & -x & 3 \\ \hline 2x-1 & 4x^3 & 4x^2 & 7x & 2 \\ & 4x^3 & -2x^2 & & \\ \hline & & -2x^2 & 7x & \\ & & -2x^2 & x & \\ \hline & & & 6x & 2 \\ & & & 6x & -3 \\ \hline & & & & 5 \end{array}$
		3	

Question	Answer	Marks	Guidance
2	**Either:** Use law of logarithm of a product or quotient Allow later if seen applied to correct terms	**M1**	The two M marks are independent.
	At least one use of the correct power law to a correct term.	**M1**	E.g. $3(5-2x)\ln 2=(x+1)\ln 4$ or $(5-2x)\ln 6=(x+1)\ln 4$ scores M0M1.
	Obtain a correct linear equation in any form, e.g. $(5 – 2x)\ln 2 = (x + 1) \ln 4 – \ln 3$	**A1**	If using invisible brackets, A1 is not allowed until their meaning becomes clear.
	Obtain $(x =)\ \dfrac{\ln 24}{\ln 16}$	**A1**	OE of the form $\dfrac{\ln a}{\ln b}$ e.g. $\dfrac{\ln 576}{\ln 256}$, Do not ISW. No working shown, no marks.
	Or: Use a correct law of indices to obtain the same base on both sides	**B1**	E.g. writing 4^{x+1} as 2^{2x+2} or 2^{5-2x} as $4^{2.5-x}$ scores B1 but $\dfrac{2^5}{2^{2x}}$ on its own scores B0.
	Obtain a correct equation in any form with a single index term involving x	**B1**	E.g. $24 = 2^{4x}$ or $3 = 2^{4x-3}$.
	Correctly take logs of an equation in any form with a single index term to obtain a linear equation	**M1**	E.g. $\ln 3=(4x-3)\ln 2$ or $\log_2 3=4x-3$.
	Obtain $(x =)\ \dfrac{\ln 24}{\ln 16}$	**A1**	OE of the form $\dfrac{\ln a}{\ln b}$ e.g. $\dfrac{\ln 576}{\ln 256}$. Do not ISW. No working shown, no marks.
	Available marks	**4**	

Question	Answer	Marks	Guidance
3(a)	$r = 6$	**B1**	SOI ISW
	$\theta = 1$	**B1**	SOI ISW $6(\cos 1 + i\sin 1)$ scores B1B1. $6e^{i}$, $6e^{1i}$ or $6e^{i1}$ score B1B1. Allow B1B1 if the correct answers follow a decimal approximation, e.g. $\theta = 1$ from $\theta = 1.0006$. Accept AWRT 1.00 and 6.00.
		2	
3(b)	Enlargement [scale] factor 2 Allow “expansion”, “scaling”, “stretch”, “dilation” or “the length is doubled” for enlargement Do not allow, e.g., “reduction” or “shrink” or “compression” for enlargement	**B1**	Need description and scale factor. Not required to state centre (0, 0) but if incorrect then B0. B0 if “stretched along the *y*-axis”
	Clockwise rotation of 1 [radians] or rotation of –1 [radians] (direction implied) or [anticlockwise] rotation of $(2\pi - 1)$ [radians]	**B1**	Need direction and amount. Not required to state centre (0, 0), but if incorrect then B0. Allow “the argument is halved”. If B0B0 scored, allow **SC** B1 for “enlargement” OE AND “rotation” OE stated. *Their* description to be considered; ignore any sketch.
		2	

Question	Answer	Marks	Guidance
4	Square $x + iy$ and equate real and imaginary parts to 3 and $-6\sqrt{2}$ respectively	*M1	Attempt to find $x^2 - y^2$ [= 3] and $(x^2 + y^2)^2$ [= 9^2]. Allow one slip anywhere.
	Obtain equations $x^2 - y^2 = 3$ and $2xy = -6\sqrt{2}$	A1	OE Allow $x^2 - y^2 = 3$ and $x^2 + y^2 = 9$ instead. Allow $2xyi = -6\sqrt{2}i$.
	Eliminate one variable and find an equation in the other	DM1	Requires two equations in two unknowns. Allow slips but the algebra must be sensible. E.g. M0 if using $x - y = \sqrt{3}$ or $x = -3\sqrt{2}y$.
	Obtain $x^4 - 3x^2 - 18 = 0$ or $y^4 + 3y^2 - 18 = 0$ or three-term equivalents	A1	Or $2x^2 = 12$ or $2y^2 = 6$ OE. A0 for a fortuitously correct equation following a sign error.
	Obtain answers $\pm\left(\sqrt{6} - \sqrt{3}i\right)$ only	A1	Needs to be in the form $x + iy$, do not accept x and y stated separately. Allow unsimplified surd forms, e.g. $\frac{\sqrt{12}}{2}i$. Ignore unsimplified equivalents coming from the complex roots of the quartic.
			SC: If working with decimal approximations to the surds, allow M1A1M1A1A0 $\left(6\sqrt{2} = 8.48...\right)$.
		5	

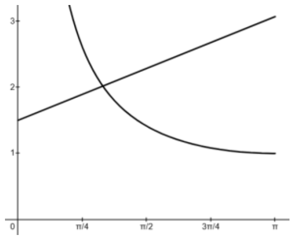
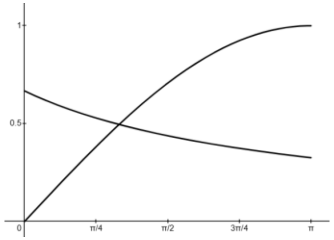
Question	Answer	Marks	Guidance
5(a)	State $\dfrac{dx}{d\theta} = 2 - 2\sec^2 2\theta$	**B1**	OE
	Use chain rule to obtain $\dfrac{d}{d\theta}\ln(\cos 2\theta) = \dfrac{\frac{d}{d\theta}(\cos 2\theta)}{\cos 2\theta}$	**M1**	E.g. $\dfrac{d}{d\theta}\ln(\cos 2\theta) = \dfrac{-2\sin 2\theta}{\cos 2\theta}$ or $\dfrac{d}{d\theta}\ln(\cos^2 2\theta) = \dfrac{-4\sin 2\theta\cos 2\theta}{\cos^2 2\theta}$. Allow an error in the coefficient but not $\dfrac{1}{\cos 2\theta}$.
	Obtain $\dfrac{dy}{d\theta} = 6 - \dfrac{4\sin 2\theta}{\cos 2\theta}$	**A1**	OE
	Use $\dfrac{dy}{dx} = \dfrac{dy}{d\theta} \div \dfrac{dx}{d\theta}$	**M1**	E.g. $\dfrac{dy}{dx} = \dfrac{6 - \left(\dfrac{4\sin 2\theta}{\cos 2\theta}\right)}{2 - 2\sec^2 2\theta}$ need not be in terms of $\tan 2\theta$ for M1. Allow M1 even if errors in both derivatives. M0 if $\dfrac{dx}{d\theta} = 2 - 2\sec^2 2\theta$ becomes $\dfrac{d\theta}{dx} = \dfrac{1}{2} - \dfrac{1}{2}\cos^2 2\theta$ before use in $\dfrac{dy}{dx}$.
	Use $\sec^2 2\theta = 1 + \tan^2 2\theta$ and $\dfrac{\sin 2\theta}{\cos 2\theta} = \tan 2\theta$ to write $\dfrac{dy}{dx}$ or $\dfrac{dx}{d\theta}$ and $\dfrac{dy}{d\theta}$ in terms of $\tan 2\theta$	**M1**	OE Independent of the second M1.
	Obtain $\dfrac{dy}{dx} = \dfrac{6 - 4\tan 2\theta}{-2\tan^2 2\theta}$	**A1**	OE ISW E.g. $\dfrac{dy}{dx} = \dfrac{2\tan 2\theta - 3}{\tan^2 2\theta}$ Must be in terms of $\tan 2\theta$. Could be two terms.

Question	Answer	Marks	Guidance
5(a)			Can start by substituting $u = 2\theta$ and work throughout in terms of u, then revert to 2θ.
		6	
5(b)	Set *their* $\frac{dy}{dx} = -1$, rearrange to a three-term quadratic in $\tan 2\theta$	***M1**	If correct, expect e.g. $\tan^2 2\theta + 2\tan 2\theta - 3 = 0$. Terms do not all need to be on the same side.
	Solve *their* three-term quadratic to obtain a value of θ or 2θ	**DM1**	Must be a quadratic with real roots in order to be equivalent work.
	Obtain $\theta = \frac{1}{8}\pi$ only within the range	**A1**	OE, e.g. $2\theta = \frac{1}{4}\pi$. Ignore solutions outside the range.
	Obtain $x = \frac{1}{4}\pi - 1$ and $y = \frac{3}{4}\pi + 2\ln\frac{\sqrt{2}}{2}$ **only** or exact equivalents	**A1**	Trig must be evaluated, e.g. $y = \frac{3}{4}\pi - \ln 2$. A0 for an answer in terms of $\tan^{-1} 1$ or 45°.
		4	

Question	Answer	Marks	Guidance
6(a)	Use the correct product rule	**M1**	I.e. $x\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\tan^{-1}3x\right)+\tan^{-1}3x\dfrac{\mathrm{d}}{\mathrm{d}x}(x)$. Allow slips but use of $\tan^{-1}\alpha=\cot\alpha$ scores M0.
	Differentiate $\tan^{-1}3x$ to obtain $\dfrac{p}{1+qx^2}$	**B1**	$q\neq 0$ Can be implied by correct form appearing in the derivative.
	Obtain $\dfrac{3x}{1+9x^2}+\tan^{-1}3x$	**A1**	OE
	Set $x=\frac{1}{3}$ and obtain $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{4}\pi+\frac{1}{2}$ only	**A1**	Or exact equivalent. Must evaluate the trigonometric terms.
		4	

Question	Answer	Marks	Guidance
6(b)	Commence integration by parts and reach $Ax^2\tan^{-1}3x \pm \int \frac{Bx^2}{D+Cx^2}\,dx$	*M1	OE
	Obtain $\frac{1}{2}x^2\tan^{-1}3x - \frac{3}{2}\int \frac{x^2}{1+9x^2}\,dx$	A1	OE
	Attempt to reduce *their* $\frac{Bx^2}{D+Cx^2}$ to $E + \frac{F}{D+Cx^2}$	DM1	
	Complete integration and obtain $\frac{1}{2}x^2\tan^{-1}3x - \frac{1}{6}x + \frac{1}{18}\tan^{-1}3x$	A1	OE
	Substitute limits correctly in an expression of the form $ax^2\tan^{-1}3x + bx + c\tan^{-1}3x$	DM1	With trigonometry evaluated. $\frac{1}{18}\left(\frac{\pi}{4}\right) - \frac{1}{18} + \frac{1}{18}\left(\frac{\pi}{4}\right)(-0+0-0)$ if correct. Condone if no evidence of use of the lower limit.
	Obtain answer $\frac{1}{36}\pi - \frac{1}{18}$	A1	Or an exact two-term equivalent. Accept as a single fraction with a two-term numerator.
		6	

Question	Answer	Marks	Guidance
7(a)	Use the correct product or quotient rule	*M1	I.e. $\dfrac{(2-\cos 3x)\dfrac{d}{dx}(3+\sin 3x)-(3+\sin 3x)\dfrac{d}{dx}(2-\cos 3x)}{(2-\cos 3x)^2}$ or $(3+3\sin 3x)\frac{d}{dx}\left((2-\cos x)^{-1}\right)+(2-\cos x)^{-1}\frac{d}{dx}(3+3\sin 3x)$ with an attempt at the derivatives. Could be using invisible brackets for M1
	Obtain the correct derivative in any form, e.g. $\dfrac{(2-\cos 3x)(3\cos 3x)-(3+\sin 3x)(3\sin 3x)}{(2-\cos 3x)^2}$	A1	Must see evidence that invisible brackets are used correctly.
	Equate *their* derivative to $-\frac{3}{2}$, and obtain a three-term quadratic in $\sin 3a$ or $\sin 3x$	DM1	Will require correct use of correct trig identity. Allow if all the working is in terms of x or they have a mixture of x and a.
	Obtain $\sin^2 3a+6\sin 3a-3=0$ with full and correct working	A1	**AG** Must be in terms of a. A0 if an error seen and not recovered in the next line.
		4	
7(b)	Find a value of a in the given interval	M1	Requires an intermediate step, e.g. $\sin 3a=-3\pm\sqrt{12}$.
	Obtain an answer, e.g. 0.161	A1	Condone if they have $x=\ldots$
	Obtain all answers 0.161, 0.886, 2.26, 2.98 and no others in the interval (9.2°, 50.8°, 129.2°, 170.8°)	A1	Treat answers in degrees as a misread. Withhold one A1 only for instances of this throughout. Ignore answers outside the given interval. Need at least 3 sf for each value in radians. Allow AWRT. Correct answer from incorrect working or no working scores 0 marks.
		3	

Question	Answer	Marks	Guidance
8(a)	Sketch $y = \operatorname{cosec} \frac{1}{2}x$ for $0 \le x \le \pi$ The horizontal level of the minimum should not look as if it is on the x-axis	**M1**	Allow both scales implied (could be using the horizontal lines for their vertical scale). Allow dashes, labelling of individual points. Minimum at $(\pi,\ 1)$, not turning upwards within the required interval. Vertical asymptote in roughly the correct place – do not allow if doubles back. Where points are plotted, no part of the graph should be lost (especially between 0 and 1).
	Sketch $y = \frac{1}{2}(x+3)$ for $0 \le x \le \pi$ and justify the given statement by clear marking of the intersect or a statement or a dotted line down to the x-axis Consistent vertical scale: e.g., y-intercept above the minimum for the trigonometric graph	**A1**	 or  Ignore anything (correct or incorrect) outside the interval. Might also see use of $\sin \frac{1}{2}x$ and $\dfrac{2}{x+3}$, in which case M1 for either (shape and key points) and A1 for correct completion.
		2	

Question	Answer	Marks	Guidance
8(b)	Calculate the values of a relevant expression or pair of expressions at $x = 1$ and $x = 1.2$ Using $2\text{cosec}\left(\frac{1}{2}x\right) - 3 = x$, 1.17 >1, 0.54 < 1.2	**M1**	Must calculate both values, and one must be correct. $\text{f}(x) = \text{cosec}\,\frac{1}{2}x - \frac{1}{2}(x+3)$ $\text{f}(1) = 0.0858\ldots > 0$ $\text{f}(1.2) = -0.328\ldots < 0$ Allow $2.0858 - 2 > 0$ and $1.77 - 2.1 < 0$. Or comparing $\text{cosec}\,\frac{1}{2}x$ and $\frac{1}{2}(x+3)$. 1 $\text{cosec}\,\frac{1}{2}x = 2.085\ldots$ $\frac{1}{2}(x+3) = 2$ $\Rightarrow$ $2.085\ldots > 2$ 1.2 $\text{cosec}\,\frac{1}{2}x = 1.771\ldots$ $\frac{1}{2}(x+3) = 2.1$ $\Rightarrow 1.771\ldots < 2.1$ Or using $1 = \frac{1}{2}(x+3)\sin\left(\frac{1}{2}x\right)$: 1 > 0.9589, 1 < 1.1857. Working in degrees is M0.
	Complete the argument correctly	**A1**	With correct values calculated.
		2	

Question	Answer	Marks	Guidance
8(c)	**EITHER:**		
	Express $x_{n+1} = 2\sin^{-1}\left(\dfrac{2}{x_n+3}\right)$ as $x = 2\sin^{-1}\left(\dfrac{2}{x+3}\right)$	**(M1**	Allow to start from $\frac{x}{2} = \sin^{-1}\left(\dfrac{2}{x+3}\right)$.
	Rearrange $x = 2\sin^{-1}\left(\dfrac{2}{x+3}\right)$ to $\operatorname{cosec}\frac{1}{2}x = \frac{1}{2}(x+3)$ with full and correct working	**A1)**	**AG**
	OR:		
	Rearrange $\operatorname{cosec}\frac{1}{2}x = \frac{1}{2}(x+3)$ to $x = 2\sin^{-1}\left(\dfrac{2}{x+3}\right)$	**(M1**	
	Express $x = 2\sin^{-1}\left(\dfrac{2}{x+3}\right)$ as $x_{n+1} = 2\sin^{-1}\left(\dfrac{2}{x_n+3}\right)$ after full and correct working	**A1)**	**AG**
	Available marks	**2**	

Question	Answer	Marks	Guidance
8(d)	Use the iterative process correctly at least once	**M1**	Use the formula once and use the answer to obtain a second value. Allow use of values rounded to 5 dp but M0 if consistently use *their* 3 dp approximations. Working in degrees is M0. Allow if they have an initial value outside the interval.
	Obtain final answer 1.037	**A1**	
	Show sufficient iterations to at least 5 dp to justify 1.037 to 3 dp or show there is a sign change in the interval (1.0365, 1.0375)	**A1**	1, 1.04720, 1.03376, 1.03755, 1.03648, 1.03678, 1.03669 1.1, 1.01915, 1.04170, 1.03530, 1.03711, 1.03660 1.2, 0.99263, 1.04933, 1.03316, 1.03772, 1.03643, 1.03679, 1.03669 $\pi/3$, 1.03376, 1.03755, 1.03648, 1.03678, 1.03669 Allow recovery after one incorrect value, truncation and allow small differences in the final digit.
		3	

Question	Answer	Marks	Guidance
9(a)	Use a correct method to form an equation for line *m*	**M1**	Using the direction vector and a point on the line. Must have a correct method for the direction vector but condone arithmetic slips.
	Obtain $\mathbf{r} = \begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$	**A1**	OE Must have $\mathbf{r} = \ldots$ or $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \ldots$ $m = \ldots$ is A0, R_m is A0, $AB = \ldots$ is A0. May use B; $\begin{pmatrix} 4 \\ 3 \\ 1 \end{pmatrix}$ in place of A and/or $\lambda \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$. Accept as column vectors, or in **i**. **j**. **k** form. Condone if the brackets are missing on the position vector.
		2	

<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td>9(b)</td><td>Express l or m in component form,
e.g. $(6+\mu, -1-\mu, -3-2\mu)$ or $(3+\lambda_1, 4-\lambda_1, 1)$ or $(4+\lambda_2, 3-\lambda_2, 1)$</td><td>B1</td><td>Can be implied if see at least two correct components for one line being stated or used in the subsequent working.</td></tr>
<tr><td></td><td>Equate pairs of components of general points on l and their m to obtain an answer for λ or μ, e.g. $\mu = -2$ or $\lambda_1 = 1$ or $\lambda_1 = 3$ if equations correct, or determine no value can be found for λ or μ if using equations for i and j</td><td>M1</td><td></td></tr>
<tr><td></td><td>Determine that all three equations are not satisfied by
showing that they cannot be solved
obtaining a contradiction, e.g. as on the table on the right even if, for example, they state ‘6 = 4’. Do require correct values
Not necessary to see a comment about the contradiction, or a comment that the lines do not intersect
Ignore any apparent conclusion for this mark</td><td>A1</td><td>Alternatives:
<table>
<tr><td>1</td><td>μ</td><td>λ_1</td><td></td><td>2</td><td>μ</td><td>λ_2</td><td></td></tr>
<tr><td>ij</td><td>–</td><td>–</td><td>$7 \neq 5$</td><td>ij</td><td>–</td><td>–</td><td>$7 \neq 5$</td></tr>
<tr><td>jk</td><td>–2</td><td>3</td><td>$6 \neq 4$</td><td>jk</td><td>–2</td><td>2</td><td>$6 \neq 4$</td></tr>
<tr><td>ik</td><td>–2</td><td>1</td><td>$3 \neq 1$</td><td>ik</td><td>–2</td><td>0</td><td>$3 \neq 1$</td></tr>
</table></td></tr>
<tr><td></td><td>Justify lines are not parallel, e.g. $\begin{pmatrix}1\\-1\\-2\end{pmatrix} \neq d\begin{pmatrix}1\\-1\\0\end{pmatrix}$, and conclude the lines are skew</td><td>A1</td><td>E.g. $\begin{pmatrix}1\\-1\\-2\end{pmatrix} \times \begin{pmatrix}1\\-1\\0\end{pmatrix} \neq 0$ or
$\begin{pmatrix}1\\-1\\-2\end{pmatrix} \cdot \begin{pmatrix}1\\-1\\0\end{pmatrix} = 1+1+0 = 2 \neq \sqrt{1^2+(-1)^2+(-2)^2}$
$\sqrt{1^2+(-1)^2+0^2} = \sqrt{6}\sqrt{2} = \sqrt{12}$
Conclusion needs to follow fully correct work.
Dependent on the three previous marks being gained.
Do not accept direction vectors are not equal, or direction vectors are different, or $\mu \neq \lambda$, or scalar product $\neq 0$, or the line equations are not multiples of each other.
It must be clearly shown which vectors are the direction vectors, e.g. by stating them or using them.</td></tr>
</table>

Question	Answer	Marks	Guidance
9(b)	SC if correct demonstration of non-parallel and nothing else, award B1		
		4	
9(c)	Express $\overrightarrow{PA}$ or $\overrightarrow{AP}$ in component form, e.g. $\pm(-3-\mu, 5+\mu, 4+2\mu)$	**B1**	If there is no parameter in *their* vector AP, the most they can score is B0M1M0M0M0A0.
	Carry out correct process for evaluating the scalar product of *their* $\overrightarrow{PA}$ and *their* $\overrightarrow{BA}$	**M1**	E.g. $-1(-3-\mu)+1(5+\mu)+0(4+2\mu)$, or $3+\mu+5+\mu$ or $8+2\mu$ if correct, or use *their* $\pm\overrightarrow{AP}$ and *their* $\pm\overrightarrow{AB}$.
	Complete the scalar product process using the moduli of their vectors and use $\cos 60° = \frac{1}{2}$ to form an equation involving a single parameter	**M1**	E.g. $\dfrac{3+\mu+5+\mu}{\sqrt{2}\sqrt{(3+\mu)^2+(-5-\mu)^2+(-4-2\mu)^2}}=\dfrac{1}{2}$ if correct.
	Form a horizontal equation involving a single parameter after correct removal of square roots and correct processing of $\frac{1}{2}$	***M1**	E.g. $\mu^2+16\mu+39=0$ OE if correct. Of correct form but need not be simplified yet.
	Solve *their* three-term quadratic and use a root to find a position vector for P	**DM1**	Correct values are $\mu=-3$ and $\mu=-13$.
	Obtain position vector $\begin{pmatrix}3\\2\\3\end{pmatrix}$ only	**A1**	Need to reject the position vector from using –13 if seen. Rejection can be implied by a single position vector for the final answer.
		6	

Question	Answer	Marks	Guidance
10(a)	State or imply the form $\dfrac{A}{x-1}+\dfrac{B}{2x-6}$	**B1**	
	Use a correct method for finding a constant	**M1**	E.g. by substituting an appropriate value or by comparing coefficients. Need to get as far as a value for one of the constants.
	Obtain $A = -1$ or $B = 3$	**A1**	
	Obtain $A = -1$ and $B = 3$	**A1**	ISW Do not need to show the conclusion as a pair of fractions.
		4	

Question	Answer	Marks	Guidance
10(b)	Separate variables correctly – seen or implied by correct structure to the integration of both sides	**B1**	I.e. $\int \ln 2y \, dy = \int \frac{x+3}{(x-1)(2x-6)} \, dx$ OE. Condone missing integral signs or missing dx, dy but not both unless implied by subsequent work.
	Obtain term $-\ln(x-1)$	**B1FT**	OE FT $A\ln(x-1)$
	Obtain term $\frac{3}{2}\ln(2x-6)$	**B1FT**	OE FT $\frac{B}{2}\ln(2x-6)$, or e.g. $\frac{B}{2}\ln(x-3)$.
	Commence integration by parts and reach $py\ln 2y \pm \int \frac{q}{y} y \, dy$	**M1**	OE Or use substitution and parts.
	Complete integration by parts to obtain $y\ln 2y - y$	**A1**	OE Might leave the answer as $\frac{1}{2}\left(\ln 2y e^{\ln 2y} - e^{\ln 2y}\right)$.
	Use $x = 4$, $y = 1$ to evaluate a constant of the correct form or as limits in a solution containing terms of the form $a\ln(x-1)$, $b\ln(2x-6)$, $cy\ln 2y$ and fy	**M1**	May have $b\ln(x-3)$ in place of $b\ln(2x-6)$. Allow the constant as a decimal. If they have taken exponentials, the constant must be a multiplier.
	Obtain correct answer in any form (with $e^{\ln 2}$, and similar terms, simplified if seen)	**A1**	ISW E.g. $y\ln 2y - y = -\ln(x-1) + \frac{3}{2}\ln(2x-6) + \ln 3 - \frac{1}{2}\ln 2 - 1$ or $y\ln 2y - y = -\ln(x-1) + \frac{3}{2}\ln(2x-6) - 0.2479\ldots$ or $y\ln 2y - y = -\ln(x-1) + \frac{3}{2}\ln(x-3) + \ln 6 - 1$ or $y\ln 2y - y = -\ln(x-1) + \frac{3}{2}\ln(x-3) + 0.7917\ldots$
		7	