Pearson Edexcel International Advanced Level

Wednesday 10 June 2026

Afternoon (Time: 1 hour 30 minutes)

Paper reference **WFM03/01A**

Mathematics

International Advanced Subsidiary/Advanced Level
Further Pure Mathematics F3
Question paper

You must have:
Answer book (sent separately)
Do not return this question paper with the answer book

Turn over

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Pearson

1. The hyperbola H has equation

$$\frac{x^2}{4} - \frac{y^2}{3} = 1$$

(a) Use calculus to show that an equation of the normal to H at the point P with coordinates $\left(2\sec\theta,\ \sqrt{3}\tan\theta\right)$ is

$$2x\tan\theta + \sqrt{3}\,y\sec\theta = k\tan\theta\sec\theta$$

where k is a constant to be determined.

(4)

The normal to H at the point $Q(4, 3)$ meets H again at the point R.

(b) Determine the coordinates of R.

(*Solutions relying entirely on calculator technology are not acceptable*.)

(4)

(Total for Question 1 is 8 marks)

2. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

(a) Use integration to show that

$$\int \frac{1}{1-x^2}\,dx = \operatorname{artanh} x + c \qquad |x| < 1$$

where c is an arbitrary constant.

(3)

The curve with equation

$$y = \operatorname{artanh} 2x - 8\arctan 4x \qquad 0 < x < \frac{1}{2}$$

has a stationary point P.

(b) Determine

(i) the exact x coordinate of P

(ii) the exact y coordinate of P, giving the answer in the form

$$\frac{1}{2}\ln\left(p + q\sqrt{3}\right) + \frac{r\pi}{3}$$

where p, q and r are integers.

(5)

(Total for Question 2 is 8 marks)

3.

$$\mathbf{M} = \begin{pmatrix} -2 & 1 & -3 \\ 0 & 3 & k \\ -2 & -2 & 3 \end{pmatrix}$$

where k is a constant.

Given that λ is an eigenvalue of the matrix $\mathbf{M}$

(a) show that

$$\lambda^3 - 4\lambda^2 + \mathrm{f}(k)\lambda + 6k + 36 = 0$$

where $\mathrm{f}(k)$ is a function to be determined.

(3)

Given that 2 is an eigenvalue of $\mathbf{M}$

(b) use the answer to part (a) to show that $k = -1$

(2)

Given that -3 is also an eigenvalue of $\mathbf{M}$

(c) write down the third eigenvalue of $\mathbf{M}$

(1)

(d) Determine an eigenvector, with integer components, which corresponds to the eigenvalue -3

(2)

(Total for Question 3 is 8 marks)

4.

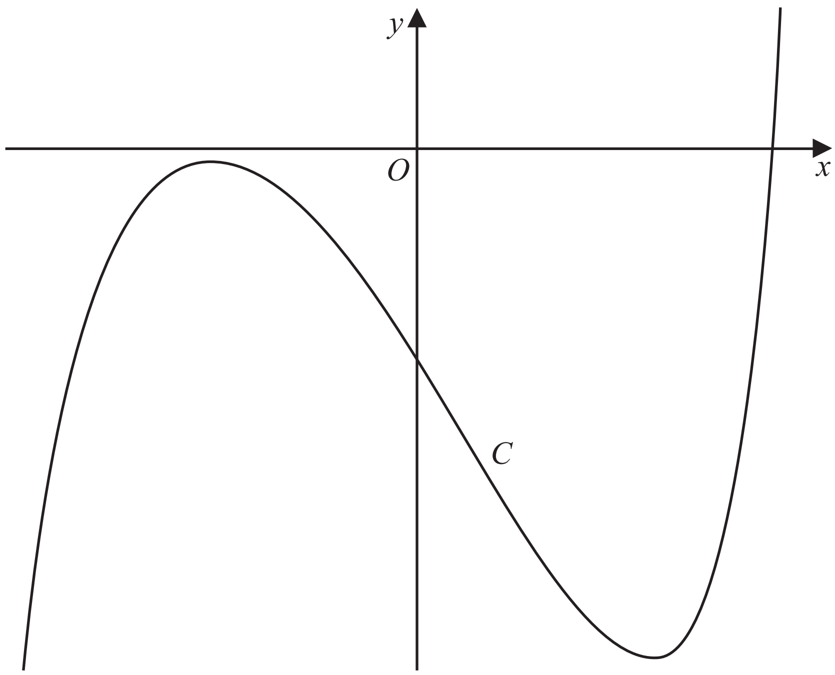

Figure 1

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

(a) Show that

$$\operatorname{arsinh} x = \ln\left(x + \sqrt{x^2 + 1}\right)$$

(3)

The curve C has equation

$$y = (\sinh x - 3)\cosh x - 6x$$

A sketch of C is shown in Figure 1.

(b) Determine the exact range of values of x for which C is decreasing, giving the answer in terms of simplified natural logarithms.

(6)

(Total for Question 4 is 9 marks)

5. Relative to a fixed origin O, the points P, Q and R have coordinates

$$P(2, 5, 0) \quad Q(-2, 6, 3) \quad R(1, 2, -1)$$

(a) (i) Determine $\overrightarrow{OP} \times \overrightarrow{OQ}$

(ii) Hence determine the exact volume of the tetrahedron $OPQR$

(3)

Given that the plane Π_1 contains the points P, Q and R

(b) determine a Cartesian equation for Π_1

(4)

The points O, Q and R are in the plane Π_2 which has equation $\mathbf{r}.(12\mathbf{i} - \mathbf{j} + 10\mathbf{k}) = 0$

Given that the point S has coordinates $(-4, -3, 2)$

(c) determine the perpendicular distance from S to Π_2, giving the answer in the form $k\sqrt{5}$ where k is a fully simplified rational number.

(2)

(Total for Question 5 is 9 marks)

6.

$$\mathbf{A} = \begin{pmatrix} p & 3 & -1 \\ 0 & 2 & -1 \\ 3 & -1 & 1 \end{pmatrix}$$

where p is a constant and $p \neq 3$

(a) Determine $\mathbf{A}^{-1}$, giving the answer in simplest form in terms of p.

(4)

The transformation represented by $\mathbf{A}$ transforms the point with position vector $2\mathbf{i} + \mathbf{j} - 5\mathbf{k}$ to the point with position vector $16\mathbf{i} + 7\mathbf{j}$

(b) Show that $p = 4$

(1)

The transformation represented by $\mathbf{A}$ transforms the point B to the point with coordinates $(2, q, -1)$ where q is a constant.

(c) Use the answer to part (a) with $p = 4$ to determine the coordinates of B. Give the answer in simplest form in terms of q.

(3)

(Total for Question 6 is 8 marks)

7. The ellipse E has equation

$$\frac{x^2}{45} + \frac{y^2}{20} = 1$$

and eccentricity e

(a) Determine

(i) the exact value of e

(ii) the equations of the directrices of E

(4)

The curve C has parametric equations

$$x = 2\cosh 2t \qquad y = 8\sinh t \qquad t > 0$$

(b) Show that a Cartesian equation for C is

$$y^2 = 16x - 32 \qquad y > 0$$

(3)

The curve C

- intersects E at the point A
- intersects a directrix of E at the point B

The arc AB of C is rotated through 2π radians about the x-axis.

(c) Use the result in part (b) and algebraic integration to determine the area of the surface generated, giving the answer in the form

$$\frac{p\pi}{3}\left(q\sqrt{q} - r\sqrt{r}\right)$$

where p, q and r are integers.

(*Solutions relying entirely on calculator technology are not acceptable.*)

(7)

(Total for Question 7 is 14 marks)

8. $$y = \sin^{n-1} x \cos x$$

(a) Show that

$$\frac{dy}{dx} = (n-1)\sin^{n-2} x - n\sin^n x$$

(2)

$$I_n = \int e^{6x} \sin^n x \, dx \qquad n \geqslant 0$$

(b) Use integration by parts to show that, for $n \geqslant 2$

$$(n^2 + 36)I_n = e^{6x} \sin^{n-1} x(6\sin x - n\cos x) + n(n-1)I_{n-2}$$

(5)

(c) Hence show that

$$\int_0^{\frac{\pi}{6}} e^{6x} \sin^2 x \, dx = \frac{1}{240}\left(e^{\pi}\left(p + q\sqrt{3}\right) + r\right)$$

where p, q and r are integers to be determined.

(4)

(Total for Question 8 is 11 marks)

TOTAL FOR PAPER IS 75 MARKS