



Mark Scheme (Final)

Summer 2026

Pearson Edexcel International Advanced Level
In Further Pure Mathematics F1 (WFM01)
Paper 01A

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

General Instructions for Marking

1. The total number of marks for the paper is 75.
2. The Edexcel Mathematics mark schemes use the following types of marks:
 - **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
 - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
 - **B** marks are unconditional accuracy marks (independent of M marks)

Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes and can be used if you are using the annotation facility on ePEN:

- bod – benefit of doubt
- ft – follow through
 - the symbol $\checkmark$ will be used for correct ft
- cao – correct answer only
- cso – correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC – special case
- oe – or equivalent (and appropriate)
- d... or dep – dependent
- indep – independent
- dp – decimal places
- sf – significant figures
- * – The answer is printed on the paper or ag- answer given

- $\square$ or d... – The second mark is dependent on gaining the first mark
4. All A marks are 'correct answer only' (cao), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.
 5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected. If you are using the annotation facility on ePEN, indicate this action by 'MR' in the body of the script.
 6. If a candidate makes more than one attempt at any question:
 - a) If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
 - b) If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

Ignore wrong working or incorrect statements following a correct answer.

Usual rules for the method mark for solving a 3 term quadratic:
(Note: There may be schemes where the below does not apply)

If no method is shown then one root must be obtained that is consistent with their equation.

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$

2. Formula

Complete attempt to use the correct formula with values for a , b and c leading to $x = \dots$ (may be unsimplified).

3. Completing the square (where $a = 1$; if $a \neq 1$ must divide by a first but allow equivalent work e.g., if a is a perfect square)

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0$, $q \neq 0$, leading to $x = \dots$

Question Number	Scheme	Notes	Marks
1	$f(z) = 4z^3 - 5z^2 + 14z + 15$ Note: A minimum acceptable solution would be: $z_1 = 1 - 2i, z_2 = 1 + 2i$ $z_1 + z_2 = 2$ $z_1 z_2 = 1 + 4 = 5$ $f(z) = (4z + 3)(z^2 - 2z + 5) \text{ [by inspection]}$ $z_3 = -\frac{3}{4}$		
	Another root is $1 + 2i$	$z = 1 + 2i$ Award B1 for sight of $1 + 2i$ anywhere in the response	B1
	$(z - (1 - 2i))(z - (1 + 2i))$ or sum = $1 + 2i + 1 - 2i = 2$, product = $(1 + 2i)(1 - 2i) = 5$ $\Rightarrow z^2 - 2z + 5$	M1: e.g. $(z \pm 1 \pm 2i)(z \pm 1 \pm 2i)$ or an attempt at the sum and product of roots. Working may be minimal. A1: Correct quadratic factor	M1 A1
	$f(z) = (4z + 3)(z^2 - 2z + 5)$ M1: Uses their 3-term quadratic expression with real coefficients to attempt a linear factor, obtaining $az + b$ $a, b \neq 0$. May be via long division, inspection or comparing coefficients. Do not accept a fully factorised cubic without seeing a valid non-calculator method for the quadratic factor at least. If the linear factor appears in the working before the quadratic factor, this probably indicates calculator use.		M1
	$z = -\frac{3}{4}$	Correct real root following correct work. Errors in long division, for example, would be A0, even if two correct roots are found. Condone work in x	A1
ALT for M1A1 M1A1	$1 + 2i + 1 - 2i + z = \frac{5}{4}$	M1: Considers the sum of all 3 roots A1: Correct equation or equivalent working	M1 A1
	e.g. $\Rightarrow z = \frac{5}{4} - 2$ $z = -\frac{3}{4}$	M1: Attempts to solves their equation, or uses equivalent work involving the sum of the three roots, reaching a value for the third root. A1: Correct real root following correct work. Note that it is possible to obtain a third root of $-\frac{3}{4}$ despite having an incorrect quadratic factor. In this case both A marks are lost. Ignore labelling, e.g. z, z_3, or may work in x.	M1 A1
(5)			
Total 5			

Question Number	Scheme		Notes	Marks
2(a)	$x = 3y - 3, xy = 36 \Rightarrow y(3y - 3) = 36 \Rightarrow 3y^2 - 3y - 36 = 0$ or $y = \frac{x}{3} + 1 \Rightarrow x\left(\frac{x}{3} + 1\right) = 36 \Rightarrow \frac{x^2}{3} + x = 36$ or $6t - \frac{18}{t} + 3 = 0 \Rightarrow 6t^2 + 3t - 18 = 0$		Eliminates x or y to form an equation in a single variable or Substitutes $x = 6t$ and $y = \frac{6}{t}$ into l and obtains an equation in t only Condone numerical slips that lead to incorrect coefficients in the 3TQ, or to e.g. $xy = 6$.	M1
	$y^2 - y - 12 = (y - 4)(y + 3) = 0 \Rightarrow y = 4, -3$ or $x^2 + 3x - 108 = (x - 9)(x + 12) = 0 \Rightarrow x = 9, -12$ or $2t^2 + t - 6 = (2t - 3)(t + 2) = 0 \Rightarrow t = \frac{3}{2}, -2$ $\Rightarrow (9, 4), (-12, -3)$		Note: Both A marks require a non-calculator solution to a 3TQ. Usual rules, but any factorisation that is inconsistent with their 3TQ is A0A0. A1: Any two of the four correct coordinates. A1: All four correct coordinates, obtained, paired correctly. Condone $x = \dots, y = \dots$ if clearly paired correctly.	A1 A1
(3)				
(b)	$\{y = 36x^{-1}\}$ $\frac{dy}{dx} = -\frac{36}{x^2}$	$\{xy = 36\}$ $y + x \frac{dy}{dx} = 0$ $\frac{dy}{dx} = -\frac{y}{x}$	$\left\{ \begin{array}{l} x = ct \\ y = \frac{c}{t} \end{array} \right\}$ $\frac{dx}{dt} = c \quad \frac{dy}{dt} = -\frac{c}{t^2}$ $\frac{dy}{dx} = -\frac{1}{t^2}$ May use $c = 6$	B1
	Differentiates the equation of the curve to obtain a correct expression for $\frac{dy}{dx}$ No calculus is B0, but allow the next two marks.			
	$m_T = -\frac{1}{4} \Rightarrow y - (-3) = -\frac{1}{4}(x - (-12))$ or $-3 = -\frac{1}{4}(-12) + c \Rightarrow c = -6$ $\{\text{leading to } y = -\frac{1}{4}x - 6\}$ $0 = -\frac{1}{4}x - 6 \Rightarrow x = \dots$		Forms equation of tangent with the numerical value for their $\frac{dy}{dx}$ and their coordinates of P (where $x < 0$) correctly placed. Must set $y = 0$ and obtain $x = \dots$ M0 for attempts at a normal.	M1
	$x = -24$		Correct x coordinate	
A1				
(3)				
Total 6				

Question Number	Scheme	Notes	Marks
3(a)	$\det \mathbf{A} = 5k(k-1) - (k+2)(k-3)$	Correct method for determinant. Allow one copying/bracketing or sign slip	M1
	$5k^2 - 5k - (k^2 - k - 6) = 5$ $4k^2 - 4k + 1 = 0 \Rightarrow (2k-1)^2 = 0 \Rightarrow k = \frac{1}{2}$	M1: Sets expression in k equal to 5, and solves a quadratic equation in k . If solving a 3TQ, usual rules apply. A1: Correct value for k	M1 A1
(3)			
(b)	$\Rightarrow \mathbf{A} = \begin{pmatrix} \frac{5}{2} & -\frac{5}{2} \\ \frac{5}{2} & -\frac{1}{2} \end{pmatrix} \Rightarrow \mathbf{A}^{-1} = \frac{1}{5} \begin{pmatrix} -\frac{1}{2} & \frac{5}{2} \\ -\frac{5}{2} & \frac{5}{2} \end{pmatrix}$ $= \frac{1}{10} \begin{pmatrix} -1 & 5 \\ -5 & 5 \end{pmatrix}$	M1: Uses their value of k to find $\mathbf{A}$ and uses an acceptable method to find $\mathbf{A}^{-1}$. Accept any matrix that is not $\mathbf{A}$ multiplied by $\frac{1}{5}$. Condone matrix $\mathbf{A}$ not being written down, but must see any matrix that is not $\mathbf{A}$ multiplied by $\frac{1}{5}$. Alternatively finds $\mathbf{A}^{-1}$ in terms of k and then substitutes their k . Both methods require using $\det \mathbf{A} = 5$. A1: Correct inverse in required form	M1 A1
	Note that in terms of k , $\mathbf{A}^{-1} = \frac{1}{4k^2 - 4k + 6} \begin{pmatrix} k-1 & 3-k \\ -k-2 & 5k \end{pmatrix}$		
(2)			
(c)	$1.3^2 \times 5 = 8.45$	8.45 or exact equivalent and no other answers given.	B1
(1)			
(d)	Note: If no working is seen, a correct answer can be given M1A1 For A1 using either method, also accept a column vector or $x = \dots, y = \dots$		
	$\frac{1}{10} \begin{pmatrix} -1 & 5 \\ -5 & 5 \end{pmatrix} \begin{pmatrix} -8 \\ 6 \end{pmatrix} = \frac{1}{10} \begin{pmatrix} 8+30 \\ 40+30 \end{pmatrix} = \frac{1}{10} \begin{pmatrix} 38 \\ 70 \end{pmatrix}$ $\Rightarrow \left(\frac{19}{5}, 7 \right)$	M1: Correct method to multiply their inverse and $\begin{pmatrix} -8 \\ 6 \end{pmatrix}$. May need to be checked if $\mathbf{A}^{-1}$ is incorrect. A1: $\left(\frac{19}{5}, 7 \right)$ or $\left(3\frac{4}{5}, 7 \right)$ or $(3.8, 7)$	M1 A1
Alt	$\frac{1}{2} \begin{pmatrix} 5 & -5 \\ 5 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -8 \\ 6 \end{pmatrix} \Rightarrow \begin{matrix} 5x - 5y = -16 \\ 5x - y = 12 \end{matrix}$ $\Rightarrow x = \frac{19}{5}, y = 7$	M1: Uses (their $\mathbf{A}$) $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -8 \\ 6 \end{pmatrix}$ appropriately to form simultaneous equations and solves A1: $\left(\frac{19}{5}, 7 \right)$ or $\left(3\frac{4}{5}, 7 \right)$ or $(3.8, 7)$	M1 A1
(2)			
Total 8			

Question Number	Scheme	Notes	Marks
4	$f(x) = \frac{3}{2}x^3 - 6\sqrt{x} + \frac{1}{x}$		
(a)	$f'(x) = \frac{9}{2}x^2 - 3x^{-\frac{1}{2}} - \frac{1}{x^2}$	M1: One term differentiated correctly A1: Fully correct differentiation (any form). Condone $-1x^{-2}$ for final term.	M1 A1
(2)			
(b)	$x_1 = 0.25 - \frac{f(0.25)}{f'(0.25)}$ $= 0.25 - \frac{\frac{3}{2}(0.25)^3 - 6\sqrt{0.25} + \frac{1}{0.25}}{\frac{9}{2}(0.25)^2 - 3(0.25)^{-\frac{1}{2}} - \frac{1}{(0.25)^2}}$ $= 0.25 - \frac{1.0234375}{-21.71875} \text{ or } 0.25 - \frac{\frac{131}{128}}{-\frac{695}{32}} \text{ or } \frac{1}{4} + \frac{131}{2780} \left\{ = \frac{413}{1390} \right\}$ $= 0.25 - (-0.04712230216...)$ $= 0.2971223022... = 0.297 \text{ (3 d.p.)}$	M1: Attempts to apply the Newton-Raphson formula with their $f'(x)$ and obtains a value. Condone a copying/sign error in $f(x)$ and/or their $f'(x)$ if intention clear. Implied by awrt 0.297. Must be working if value or $f'(x)$ incorrect. Allow as minimum $x_0 = 0.25, x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} \Rightarrow x_1 = ...$ A1: awrt 0.297. Must be decimal. Note: actual root is 0.305 or 0.3054852282... isw if x_2 is attempted, etc.	M1 A1
(2)			
(c)	$f(1.65) = -0.3628914411...$ $f(1.7) = 0.1346924079...$	Attempts both $f(1.65)$ and $f(1.7)$ with at least one correct to 2 d.p.	M1
e.g., $\frac{\beta - 1.65}{1.7 - \beta} = \frac{0.3628914411...}{0.1346924079...} \Rightarrow \beta = ...$ May see $\frac{1.7 - \beta}{0.134...}$ or $\frac{\beta - 1.65}{0.362...} = \frac{1.7 - 1.65}{0.362... + 0.134...}$ Forms a correct equation in β / x with their $f(1.65)$ and $f(1.7)$ values and solves. If e.g., A is used for $\beta - 1.65$ then $1.65 + A$ must be seen later Implied by awrt 1.686. If value incorrect must see an equation but allow $\frac{1.65(0.134...) - 1.7(-0.362...)}{0.134... - (-0.362...)}$ from use of $\frac{af(b) - bf(a)}{f(b) - f(a)}$. If a correct interpolation formula is quoted, allow slips in the substitution if the intention is clear.		M1	
SC for M1M1: Some candidates are not evaluating $f(1.65)$ and $f(1.7)$. Award M1M1 if $f(1.65)$ and $f(1.7)$ are used correctly within a correct interpolation formula. For example:			

	1) $\frac{1.7 - \beta}{\beta - 1.65} = \frac{\frac{3}{2}(1.7)^3 - 6(1.7)^{1/2} + (1.7)^{-1}}{-\left[\frac{3}{2}(1.65)^3 - 6(1.65)^{1/2} + (1.65)^{-1}\right]} = 0.3711... \Rightarrow \beta = 1.686$. This scores M1M1A1. 2) $\frac{1.7 - \beta}{\beta - 1.65} = 0.3711... \Rightarrow \beta = ...$ This scores M1M1, and if their solution β is correct, also scores A1. 3) As a minimal solution: $\frac{x - 1.65}{ f(1.65) } = \frac{1.7 - x}{ f(1.7) } \Rightarrow x = 1.686$ This scores M1M1A1. The correct answer implies that correct values have been used for $f(1.65)$ and $f(1.7)$.		
	$\beta = 1.686465356... = 1.686$ (3 d.p.)	Awrt 1.686. Allow “x=” (Actual root is 1.687 or 1.686860129...)	A1
Alt	$y - (-0.362...[\text{or } 0.134...]) = \frac{0.134... - (-0.362...)}{1.7 - 1.65} (x - (1.65[\text{or } 1.7])) \Rightarrow y = 0, x = ...$ M1: As main scheme M1: Forms correct straight line equation, sets $y = 0$ and solves		M1 M1
	$\beta = 1.686465356... = 1.686$ (3 d.p.)	Awrt 1.686	A1
(3)			
Total 7			

Question Number	Scheme	Notes	Marks
5	$3z + iw = -1 + i \quad kz - w = -9i$		
(a)	$w = kz + 9i \Rightarrow 3z + i(kz + 9i) = -1 + i$; or $kz - w = -9i \Rightarrow ikz - iw = 9 \Rightarrow ikz - (-1 + i - 3z) = 9$ $\Rightarrow z(3 + ki) = 8 + i \Rightarrow z = \frac{8 + i}{3 + ki}$	Uses appropriate algebra to eliminate w and reaches $z = \frac{a + ib}{c + id} = f(k)$	M1
	$z = \frac{8 + i}{3 + ki} \times \frac{3 - ki}{3 - ki}$	Appropriate multiplier seen. Condone “ $\times(3 - ki)$ ” if subsequently both numerator and denominator are multiplied.	dM1
	$z = \frac{24 + k + (3 - 8k)i}{9 + k^2} *$	Fully correct proof. Condone invisible brackets if recovered.	A1*
Alt Sim equations	$z = a + bi, w = c + di$ Substitute into given equations: $3(a + bi) + i(c + di) = -1 + i$ $k(a + bi) - (c + di) = -9i$ Equate real and imaginary parts, leading to: $3a - d = -1, 3b + c = 1$ $ka - c = 0, kb - d = -9$ Eliminate c and d leading to: $a = \frac{24 + k}{9 + k^2}$ and $b = \frac{3 - 8k}{9 + k^2}$ So $z = \frac{24 + k + (3 - 8k)i}{9 + k^2} *$	M1: Obtains four simultaneous equations in the real and imaginary coefficients of z and w dM1: Eliminates real and imaginary coefficients of w A1*: Fully correct proof.	
(3)			
(b)	$k = 2 \Rightarrow z = \frac{26 - 13i}{13} = 2 - i$ $\Rightarrow w = \dots$	Substitutes $k = 2$ to find z and then substitutes to find w . If they substitute $k = 2$ into simultaneous equations, they must reach $w = \dots$	M1
	$w = 2(2 - i) + 9i = 4 + 7i *$	Correct proof with sufficient working	A1*
(2)			
(c)(i)	$\frac{ w }{ z } = \frac{\sqrt{4^2 + 7^2}}{\sqrt{2^2 + 1^2}} = \frac{\sqrt{65}}{\sqrt{5}} = \sqrt{13}$; or $\frac{ w }{ z } = \frac{ w }{ z } = \frac{ 4 + 7i }{ 2 - i } = \left \frac{1}{5} + \frac{18}{5}i \right = \sqrt{\left(\frac{1}{5}\right)^2 + \left(\frac{18}{5}\right)^2} = \sqrt{13}$	M1: Correct use of modulus formula. Award when a correct numerical expression is seen for either modulus. Note: may see $\sqrt{zz^*}$ or $\sqrt{ww^*}$ A1: $\sqrt{13}$ only	M1 A1
(c)(ii)	$\arg(z + w) = \arg(6 + 6i)$ $= \arctan(1) = \frac{\pi}{4}$	$\frac{\pi}{4}$ or $45\{^\circ\}$ Isw if $\frac{\pi}{4}$ is written as a decimal (rounded or truncated to 0.78 or 0.79 to 2 d.p.), but not if a correct answer is followed by e.g. $-\frac{\pi}{4}, \frac{3\pi}{4}, -45^\circ, 135^\circ$	B1
(3)			
Total 8			

Question Number	Scheme		Notes	Marks
6(a)	$n = 1 \Rightarrow \text{LHS} = \begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix}^1 = \begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix}$ $\text{RHS} = \begin{pmatrix} 2(1)+1 & -2(1) \\ 2(1) & 1-2(1) \end{pmatrix} = \begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix}$		Shows that LHS and RHS = $\begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix}$ or A for $n = 1$ At minimum, condone only substitution into RHS, reaching $\begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix}$ and statement “true” o.e. Note: it is possible to use $n = 0$ in the basis case. In this case they would prove LHS=RHS=I Note: the conclusion for the basis case may be seen as a part of the final conclusion of proof.	B1
	Assume true for $n = k$, i.e., $\begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix}^k = \begin{pmatrix} 2k+1 & -2k \\ 2k & 1-2k \end{pmatrix}$			
	$\begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix}^{k+1} = \begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix}^k \begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix} = \begin{pmatrix} 2k+1 & -2k \\ 2k & 1-2k \end{pmatrix} \begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix} \text{ or } \begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix} \begin{pmatrix} 2k+1 & -2k \\ 2k & 1-2k \end{pmatrix}$ Forms appropriate product of matrices for $\mathbf{A}^{k+1}$			M1
	$= \begin{pmatrix} 6k+3-4k & -4k-2+2k \\ 6k+2-4k & -4k-1+2k \end{pmatrix} \text{ or } \begin{pmatrix} 6k+3-4k & -6k-2+4k \\ 4k+2-2k & -4k-1+2k \end{pmatrix} \Rightarrow \begin{pmatrix} 2k+3 & -2k-2 \\ 2k+2 & -2k-1 \end{pmatrix}$ dM1: Correct method for multiplication. Look for 3 correct elements unsimplified. A1: Correct simplified matrix with k terms collected. This must be seen, not implied, before being written in terms of $k+1$.			dM1 A1
	$= \begin{pmatrix} 2(k+1)+1 & -2(k+1) \\ 2(k+1) & 1-2(k+1) \end{pmatrix}$ True for $n = 1$, true for $n = k+1$ when assumed true for $n = k$, hence true for $n \in \mathbb{Z}^+$		Correct working leading to correct matrix in terms of $k+1$ and suitable conclusion. The conclusion must have 3 parts: 1) “True for $n = 1$”, but this need not be repeated if seen earlier. 2) True for $n = k \Rightarrow$ true for $n = k+1$, but if “Assume true for $n = k$” is seen earlier, then this need not be repeated, i.e. can conclude with “Hence true for $n = k+1$” o.e. 3) True for $n \in \mathbb{Z}^+$, but accept any of these: true for $n \in \mathbb{Z}$; true for all n; true for all integers; true for all numbers; true for $n \in \mathbb{N}$; true for all; always true. A1 can follow B0. A0 if work is in terms of n throughout.	A1
	Note: A meet in the middle approach is possible. For example, reaching $\begin{pmatrix} 2k+3 & -2k-2 \\ 2k+2 & -2k-1 \end{pmatrix}$ for dM1 A1 and expanding $\begin{pmatrix} 2(k+1)+1 & -2(k+1) \\ 2(k+1) & 1-2(k+1) \end{pmatrix}$ to reach the same.			

(b)	$\mathbf{A}^5 = \begin{pmatrix} 11 & -10 \\ 10 & -9 \end{pmatrix}$	Correct $\mathbf{A}^5$, but also give B1 if their calculation of $\mathbf{A}^5$ is unsimplified and/or embedded in their calculation of $\mathbf{C}$.	B1
	$\mathbf{C} = \mathbf{A}^5 - \mathbf{A}\mathbf{B} + 7\mathbf{I} = \begin{pmatrix} 11 & -10 \\ 10 & -9 \end{pmatrix} - \begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix} \begin{pmatrix} p & -2 \\ -4 & 2 \end{pmatrix} + \begin{pmatrix} 7 & 0 \\ 0 & 7 \end{pmatrix}$ $= \begin{pmatrix} 11 & -10 \\ 10 & -9 \end{pmatrix} - \begin{pmatrix} 3p+8 & -10 \\ 2p+4 & -6 \end{pmatrix} + \begin{pmatrix} 7 & 0 \\ 0 & 7 \end{pmatrix} = \dots$ A full attempt at $\mathbf{C}$ with their $\mathbf{A}^5$ including correct method of multiplication for $\mathbf{A}\mathbf{B}$ (with 3 correct elements unsimplified). Must reach a single matrix $\mathbf{C}$ in terms of p.		M1
	$\begin{pmatrix} 10-3p & 0 \\ 6-2p & 4 \end{pmatrix}$	Correct matrix	A1
(3)			
(c)	$\left\{ p=3 \Rightarrow \mathbf{C} = \begin{pmatrix} 1 & 0 \\ 0 & 4 \end{pmatrix} \right\}$ Stretch of scale factor 4 parallel to the y-axis o.e.	M1: If a matrix $\mathbf{C}$ is written down, it must be of the form $\mathbf{C} = \begin{pmatrix} 1 & 0 \\ 0 & q \end{pmatrix}, q \neq 1$. States “stretch”. Condone “enlarge”, but not “multiply”. Must be a single transformation. M0 if alternatives also given. A1ft: Fully correct description. Requires “scale factor 4”, “4 times” o.e., not e.g. “by 4 units”. Accept “y-direction” or “vertically”. FT on their $q \neq 1$	M1 A1ft
(2)			
Total 10			

Question Number	Scheme	Notes	Marks
7(a)	$2x^2 - 3x + 5 = 0 \Rightarrow \alpha + \beta = \frac{3}{2}, \alpha\beta = \frac{5}{2}$	Either correct value. Award this mark if the value(s) are only seen embedded in calculations.	B1
	$(\alpha + 1)^3 (\beta + 1)^3 = (\alpha\beta + \alpha + \beta + 1)^3$	States or uses $(\alpha + 1)^3 (\beta + 1)^3 = (\alpha\beta + \alpha + \beta + 1)^3$	M1
	$= \left(\frac{5}{2} + \frac{3}{2} + 1\right)^3 = 5^3 = 125^*$	Achieves result via correct work with substitution seen	A1*
Alt Working backwards	$2x^2 - 3x + 5 = 0 \Rightarrow \alpha + \beta = \frac{3}{2}, \alpha\beta = \frac{5}{2}$	As above	B1
	$(\alpha + 1)^3 (\beta + 1)^3 = 125$ $\Rightarrow (\alpha + 1)(\beta + 1) = 5$ $\alpha\beta + \alpha + \beta + 1 = 5$	Cube roots and expands brackets to give an expression in terms of $\alpha + \beta$ and $\alpha\beta$	M1
	$\frac{5}{2} + \frac{3}{2} + 1 = 5 \checkmark$	Verification that LHS=RHS. Must be some minimal conclusion, e.g. $\checkmark$, “shown”, “QED”, “proved”, etc.	A1*
Alt Changing variable	Let $z = x + 1 \Rightarrow x = z - 1$	$x = z - 1$	B1
	$2x^2 - 3x + 5 = 0$ $\Rightarrow 2(z - 1)^2 - 3(z - 1) + 5 = 0$ $\Rightarrow 2z^2 - 7z + 10 = 0$	Substitutes $x = z - 1$ and obtains 3TQ in z	M1
	Product of roots = $\frac{10}{2} = 5$ $\Rightarrow (\alpha + 1)^3 (\beta + 1)^3 \{= 5^3\} = 125^*$	Correct completion	A1*
Alt Expanding brackets for the last two marks	It is possible (but tricky) to complete this proof by expanding brackets. $(\alpha + 1)^3 (\beta + 1)^3 = (\alpha^3 + 3\alpha^2 + 3\alpha + 1)(\beta^3 + 3\beta^2 + 3\beta + 1)$ $= (\alpha\beta)^3 + (\alpha^3 + \beta^3) + 3(\alpha\beta)^2(\alpha + \beta) + 3\alpha\beta(\alpha^2 + \beta^2) + 9(\alpha\beta)^2$ $+ 9\alpha\beta(\alpha + \beta) + 9\alpha\beta + 3(\alpha^2 + \beta^2) + 3(\alpha + \beta) + 1$ Then using $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$ and $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$: $= (\alpha\beta)^3 + (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta) + 3(\alpha\beta)^2(\alpha + \beta) + 3\alpha\beta[(\alpha + \beta)^2 - 2\alpha\beta] + 9(\alpha\beta)^2$ $+ 9\alpha\beta(\alpha + \beta) + 9\alpha\beta + 3[(\alpha + \beta)^2 - 2\alpha\beta] + 3(\alpha + \beta) + 1$ M1: Reaches an expression they can evaluate, involving terms that they have been given or have evaluated using correct identities. A1*: Substitutes values (e.g. for $\alpha\beta$ and $(\alpha + \beta)$) and reaches 125 with no errors.		

(3)

(b)	$\text{new sum} = \frac{(\alpha+1)^3 + (\beta+1)^3}{(\alpha+1)^3 (\beta+1)^3} \quad \text{new product} = \frac{1}{(\alpha+1)^3 (\beta+1)^3}$		
	$\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta) \left\{ = -\frac{63}{8} \right\} \text{ or}$ $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta \left\{ = -\frac{11}{4} \right\} \text{ or}$ $(\alpha+1)^3 + (\beta+1)^3 = (\alpha+1+\beta+1)^3 - 3(\alpha+1)(\beta+1)(\alpha+1+\beta+1)$	Evaluates, states or uses at least one of these identities. May be seen or implied by working.	B1 (M1 on epen)
	Example 1 (expanding brackets): $(\alpha+1)^3 + (\beta+1)^3$ $= \alpha^3 + 3\alpha^2 + 3\alpha + 1 + \beta^3 + 3\beta^2 + 3\beta + 1$ $= \alpha^3 + \beta^3 + 3(\alpha^2 + \beta^2) + 3(\alpha + \beta) + 2$ $= (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta) + 3((\alpha + \beta)^2 - 2\alpha\beta) + 3(\alpha + \beta) + 2$	Manipulates $(\alpha+1)^3 + (\beta+1)^3$ to reach an expression involving only terms that have been evaluated using correct identities. Accept equivalents forms and condone sign errors and slips with coefficients only. Be aware that substitution of values may take place alongside the algebraic manipulation.	M1
	Example 2 (sum of two cubes): $(\alpha+1)^3 + (\beta+1)^3$ $= (\alpha + \beta + 2)(\alpha^2 + \beta^2 - \alpha\beta + \alpha + \beta + 1)$ $= (\alpha + \beta + 2)((\alpha + \beta)^2 - 3\alpha\beta + \alpha + \beta + 1)$		
	Example 3 (working in $\alpha+1$ and $\beta+1$): $(\alpha+1)^3 + (\beta+1)^3 = (\alpha+1+\beta+1)^3 - 3(\alpha+1)(\beta+1)(\alpha+1+\beta+1)$		
	E.g. using line 4 of Example 1: $\left(\frac{3}{2}\right)^3 - 3\left(\frac{5}{2}\right)\left(\frac{3}{2}\right) + 3\left(\left(\frac{3}{2}\right)^2 - 2\left(\frac{5}{2}\right)\right) + 3\left(\frac{3}{2}\right) + 2 = -\frac{77}{8}$ Or line 3: $\left(-\frac{63}{8}\right) + 3\left(-\frac{11}{4}\right) + 3\left(\frac{3}{2}\right) + 2 = -\frac{77}{8}$ For Example 2: $\left(\frac{3}{2} + 2\right)\left(\left(\frac{3}{2}\right)^2 - 3\left(\frac{5}{2}\right) + \frac{3}{2} + 1\right) = -\frac{77}{8}$ For Example 3: $\left(\frac{3}{2} + 2\right)^3 - 3(5)\left(\frac{3}{2} + 2\right) = -\frac{77}{8}$ $\left\{ \text{new sum} = \frac{"-77/8"}{125} = -\frac{77}{1000} \right\}$	Substitutes all relevant values correctly and reaches $-\frac{77}{8}$ o.e. for the numerator, or $-\frac{77}{1000}$ o.e. for the new sum. Note that $(\alpha^3 + \beta^3)$, $(\alpha^2 + \beta^2)$ and/or $(\alpha+1)(\beta+1)$ may have been evaluated in part (a).	A1 (M1 on epen)
	$\text{new product} = \frac{1}{125} \Rightarrow x^2 + \frac{"77/8"}{125}x + \frac{1}{125} \{=0\}$	Forms a new quadratic correctly for their new sum and the product of $\frac{1}{125}$	M1
	$1000x^2 + 77x + 8 = 0$	Any correct equation with integer coefficients	A1
(5)			
Total 8			

Question Number	Scheme		Notes	Marks
8(a)	$n = 1 \Rightarrow \text{LHS} = \sum_{r=1}^1 r^3 = 1^3 = 1$ $\text{RHS} = \frac{1}{4} \times 1^2 \times (1+1)^2 = \frac{1}{4} \times 1 \times 2^2 = 1$		Shows that LHS and RHS =1 for $n = 1$. Must be some substitution into RHS. Gives conclusion “true”, “shown”, “proved”, “✓” o.e. Note: the conclusion for the basis case may be seen as a part of the final conclusion of proof.	B1
	Assume true for $n = k$, i.e., $\sum_{r=1}^k r^3 = \frac{1}{4} k^2 (k+1)^2$			
	$\sum_{r=1}^{k+1} r^3 = \frac{1}{4} k^2 (k+1)^2 + (k+1)^3$		Uses $\sum_{r=1}^{k+1} r^3 = \left(\sum_{r=1}^k r^3 \right) + (k+1) \text{th term}$	M1
	$= (k+1)^2 \left(\frac{1}{4} k^2 + k + 1 \right) = \frac{1}{4} (k+1)^2 (k^2 + 4k + 4)$		Reaches $\frac{1}{4} (k+1)^2$ (quadratic in k) Must reach this intermediate step; it cannot be implied by $\frac{1}{4} (k+1)^2 (k+2)^2$	dM1
	$= \frac{1}{4} (k+1)^2 (k+2)^2 \text{ or } \frac{1}{4} (k+1)^2 ((k+1)+1)^2$		Either expression	A1
	True for $n = 1$, true for $n = k + 1$ when assumed true for $n = k$, hence true for $n \in \mathbb{Z}^+$	Fully correct proof with correct conclusion. The conclusion must have 3 parts: 1) “True for $n = 1$ ”, but this need not be repeated if seen earlier. 2) True for $n = k \Rightarrow$ true for $n = k + 1$, but if “Assume true for $n = k$ ” is seen earlier, then this need not be repeated, i.e. can conclude with “Hence true for $n = k + 1$ ” o.e. 3) True for $n \in \mathbb{Z}^+$, but accept any of these: true for $n \in \mathbb{Z}$; true for all n ; true for all integers; true for all numbers; true for $n \in \mathbb{N}$; true for all; always true. A1 can follow B0. A0 if work is in terms of n throughout.		A1
	Note: A meet in the middle approach is possible. For example, after reaching $\sum_{r=1}^{k+1} r^3 = \frac{1}{4} k^2 (k+1)^2 + (k+1)^3$ for M1, expanding brackets to give $\frac{1}{4} k^4 + \frac{3}{2} k^3 + \frac{13}{4} k^2 + 3k + 1$. They must then expand the target expression $\frac{1}{4} (k+1)^2 (k+2)^2$ to a quartic (dM1) to reach the same expression (A1), followed by conclusion as above (A1).			
(5)				

(b)	$\sum_{r=2n}^{4n-1} r^3 = \frac{1}{4}(4n)^2(4n-1)^2 - \frac{1}{4}(2n)^2(2n-1)^2$	Attempts $f(4n-1) - f(2n-1)$	M1
	$\sum_{r=1}^n 1 = n$	Uses $\sum_{r=1}^n 1 = n$	B1
	$4n^2(16n^2 - 8n + 1) - n^2(4n^2 - 4n + 1) = 1363n^2$ $\Rightarrow 60n^2 - 28n - 1360 = 0$ $\Rightarrow 15n^2 - 7n - 340 = 0$ $\Rightarrow (15n + 68)(n - 5) = 0 \Rightarrow n = 5$	M1: Reaches 3TQ in n that arises from at least one application of the sum of cubes formula, and solves (usual rules). Alternatively, solves a quartic to reach a value for n . In both cases, must reach a positive value of n . A1: $n = 5$ only	M1 A1
(4)			
Total 9			

Question Number	Scheme	Notes	Marks
9	$P(3p^2, 6p) \quad Q(3q^2, 6q)$		
(a)	$y = \sqrt{12}x^{\frac{1}{2}}$ $\frac{dy}{dx} = \frac{\sqrt{12}}{2}x^{-\frac{1}{2}} = \frac{\sqrt{3}}{\sqrt{3p^2}} = \frac{1}{p}$ $y^2 = 12x$ $2y \frac{dy}{dx} = 12$ $\frac{dy}{dx} = \frac{6}{y} = \frac{1}{p}$ Uses calculus to obtain $\frac{dy}{dx} = \frac{1}{p}$. This may be implied by subsequent work.	$y = 2at \quad x = at^2$ $\frac{dy}{dt} = 2a \quad \frac{dx}{dt} = 2at$ $\frac{dy}{dx} = \frac{1}{t} = \frac{1}{p}$ May use $a = 3$ and/or $t = p$	B1
	$y - 6p = -p(x - 3p^2) \quad \text{or}$ $6p = -p \times 3p^2 + c \Rightarrow c = 6p + 3p^3 \Rightarrow y = -px + 6p + 3p^3$	Forms equation of line with a changed gradient in terms of p and places coordinates correctly	M1
	$y + px = 6p + 3p^3 *$	Correct proof with evidence of calculus. Note that B0M1A1 is not possible.	A1*
(3)			
(b) Using equation of line PQ	Note: There are many variations on the below, e.g. using $(3, 0)$ to form the equation of the line (first M1), then using $(3p^2, 6p)$ or $(3q^2, 6q)$ (second M1)		
	$\text{gradient } PQ = \frac{6p - 6q}{3p^2 - 3q^2} \quad \left\{ = \frac{6(p - q)}{3(p + q)(p - q)} = \frac{2}{p + q} \right\}$	A correct expression for the gradient of the chord	B1
	$\text{e.g. } y - 6p = \frac{2}{p + q}(x - 3p^2) \quad \text{or}$ $6p = \frac{2}{p + q} \times 3p^2 + c \Rightarrow c = 6p - \frac{6p^2}{p + q} \Rightarrow y = \frac{2}{p + q}x + 6p - \frac{6p^2}{p + q}$	Forms equation of line with their chord gradient in terms of p and q and places coordinates correctly. The example uses $y - y_1 = m(x - x_1)$ with $(x_1, y_1) = (3p^2, 6p)$, but $(3, 0)$ or $(3q^2, 6q)$ could be used instead.	M1
	$-6p = \frac{6 - 6p^2}{p + q}$	Uses $(3, 0)$ with their line to form an equation in p and q	M1
	$p(p + q) = p^2 - 1 \Rightarrow p^2 + pq = p^2 - 1 \Rightarrow pq = -1 *$	Fully correct proof with sufficient working	A1*
	$\textbf{Note:}$ If the gradient is not simplified you may see e.g., $-6p = \frac{6p - 6q}{3p^2 - 3q^2}(3 - 3p^2) \Rightarrow -18p^3 + 18pq^2 = 18p - 18q - 18p^3 + 18p^2q$ $\Rightarrow -pq(p - q) = p - q \Rightarrow pq = -1 *$		
(4)			
Alt 1 Using gradients of PF and QF	$\text{grad } PF = \frac{6p}{3p^2 - 3}, \text{grad } QF = \frac{6q}{3q^2 - 3}$	A correct expression for the gradient of PF or QF	B1
		Correct expressions for gradients of both PF and QF	M1
	$\frac{6p}{3p^2 - 3} = \frac{6q}{3q^2 - 3}$	Equates to form an equation in p and q	M1
	$pq^2 - p = p^2q - q$ $-p + q = p^2q - pq^2$ $-(p - q) = pq(p - q) \Rightarrow pq = -1 *$	Fully correct proof with sufficient working	A1*

Alt 2 Similar triangles	$\frac{3p^2-3}{3-3q^2} = \frac{6p}{-6q}$		Finds either the ratio of the widths $\frac{3p^2-3}{3-3q^2}$ or the heights $\frac{6p}{-6q}$	B1
			Finds both the ratio of the widths $\frac{3p^2-3}{3-3q^2}$ and the heights $\frac{6p}{-6q}$	M1
	$\frac{p^2-1}{1-q^2} = \frac{p}{-q}$		Equates to form an equation in p and q	M1
	$-p^2q + q = p - pq^2$ $-pq(p-q) = p - q$ $pq = -1^*$		Fully correct proof with sufficient working	A1*
Alt 3 Using gradients of PQ and PF	gradient $PQ = \frac{6p-6q}{3p^2-3q^2}$, gradient $PF = \frac{6p}{3p^2-3}$		A correct expression for the gradient of PQ or PF	B1
			Correct expressions for the gradients of both PQ and PF	M1
	$\frac{6p-6q}{3p^2-3q^2} = \frac{6p}{3p^2-3}$		Equates to form an equation in p and q	M1
	$\Rightarrow \frac{1}{p+q} = \frac{p}{p^2-1} \Rightarrow p^2-1 = p^2+pq \Rightarrow pq = -1^*$		Fully correct proof with sufficient working	A1*
Alt 4 Using lengths: $PQ=PF+QF$	$PF = 3 + 3p^2$ $QF = 3 + 3q^2$ $PQ = \sqrt{(3p^2-3q^2)^2 + (6p-6q)^2}$		Correct expression for one of the lengths PQ , PF or QF , or the square of any of these	B1
			Attempts all three lengths, or their squares	M1
	e.g. $\sqrt{(3p^2-3q^2)^2 + (6p-6q)^2} = 3(p^2+q^2)+6$		Uses $PQ = PF + QF$ to form an equation in p and q	M1
	Completes the proof with no errors $(3p^2-3q^2)^2 + (6p-6q)^2 = (3(p^2+q^2)+6)^2$ Multiplying out and dividing both sides by 9 leads to: $p^4 - 2(pq)^2 + q^4 + 4p^2 - 8(pq) + 4q^2 = p^4 + 2(pq)^2 + q^4 + 4p^2 + 4q^2 + 4$ $\Rightarrow (pq)^2 + 2(pq) + 1 = 0 \Rightarrow (pq+1)^2 = 0 \Rightarrow pq = -1^*$			A1*
Alt 5 Sim equations	Line PQ has equation $y = mx + c$ Sub in $(3p^2, 6p), (3q^2, 6q), (3, 0)$: $6p = 3mp^2 + c \quad (1)$ $6q = 3mq^2 + c \quad (2)$ $0 = 3m + c \quad (3)$		Forms 3 simultaneous equations in p, q, m, c	B1
	$(1) - (3) \Rightarrow 6p = 3m(p^2 - 1) \quad (4)$ $(2) - (3) \Rightarrow 6q = 3m(q^2 - 1) \quad (5)$		Eliminates c or m	M1
	$\frac{6p}{p^2-1} = \frac{6q}{q^2-1}$		Eliminates both c and m	M1
	$pq^2 - p = p^2q - q$ $-p + q = p^2q - pq^2$ $-(p-q) = pq(p-q)$		Fully correct proof with sufficient working	A1*

	$pq = -1^*$		
9(c) Way 1	$y + px = 6p + 3p^3, \quad y + qx = 6q + 3q^3$ $\Rightarrow 6p + 3p^3 - px = 6q + 3q^3 - qx$	M1: Eliminates y without sign errors – must form a correct equation.	M1
	$6(p - q) + 3(p^3 - q^3) = (p - q)x$ $x = 6 + 3(p^2 + pq + q^2)$ $= 3 + 3(p^2 + q^2)$ $= 3 + 3(p + q)^2 - 6pq$ $= 3(p + q)^2 + 9$	M1: Uses appropriate algebra with sign/coefficient errors only to reach an expression for x of the correct form A1: Correct expression	M1 A1
	$y = 6p + 3p^3 - p(3(p + q)^2 + 9)$ $= 6p + 3p^3 - 3p^3 - 6p^2q - 3pq^2 - 9p$ $= -3p - 3pq(2p + q)$ $= -3p + 6p + 3q$ $= 3(p + q)$	dM1: Substitutes their expression for x of the correct form to obtain an expression for y of the correct form condoning sign/coefficient errors only A1: Correct expression	dM1 A1
(c) Way 2	$y + px = 6p + 3p^3, \quad y + qx = 6q + 3q^3$ $\frac{6p + 3p^3 - y}{p} = \frac{6q + 3q^3 - y}{q}$	M1: Eliminates x without sign errors – must form a correct equation.	M1
	$6pq + 3p^3q - qy = 6pq + 3pq^3 - pq$ $(p - q)y = 3pq(q^2 - p^2)$ $(p - q)y = 3(p^2 - q^2)$ $y = 3(p + q)$	M1: Uses appropriate algebra with sign/coefficient errors only to reach an expression for y of the correct form A1: Correct expression	M1 A1
	$x = \frac{6p + 3p^3 - 3(p + q)}{p}$ $= 6 + 3p^2 + \frac{3pq}{p}(p + q)$ $= 6 + 3p^2 + 3pq + 3q^2$ $= 9 + 3(p^2 + 2pq + q^2)$ $= 3(p + q)^2 + 9$	dM1: Substitutes their expression for y of the correct form to obtain an expression for x of the correct form condoning sign/coefficient errors only A1: Correct expression	dM1 A1
(5)			
(d)	$P(12, 12)$ $\Rightarrow p = 2, q = -\frac{1}{2}, p + q = \frac{3}{2}$ $x_R = 3\left(\frac{3}{2}\right)^2 + 9, y_R = 3\left(\frac{3}{2}\right)$ $\Rightarrow R\left(\frac{63}{4}, \frac{9}{2}\right)$	M1: Correct method to find one coordinate of R . Substitutes $p = 2, q = -\frac{1}{2}$ or $p + q = \frac{3}{2}$ into an expression for x or y from part (c). A1: Correct coordinates of R in simplest form, i.e. $\left(\frac{63}{4}, \frac{9}{2}\right)$ or $\left(15\frac{3}{4}, 4\frac{1}{2}\right)$ or $(15.75, 4.5)$ accept $x = \dots, y = \dots$	M1 A1
(2)			
Total 14			
PAPER TOTAL: 75			