Mark Scheme (Results)

Summer 2026

Pearson Edexcel International Advanced Level in Pure Mathematics
WMA13/01

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Summer 2026
Question Paper Log Number PXXXXA
Publication Code WMA13_01_2606_MS

EDEXCEL IAL MATHEMATICS
General Instructions for Marking

1. The total number of marks for the paper is 75

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: Method marks are awarded for 'knowing a method and completing an attempt to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

   These are some of the traditional marking abbreviations that will appear in the mark schemes.

   - ft – follow through
   - cao – correct answer only
   - cso - correct solution only. There must be no clear errors in this part of the question to obtain this mark
   - isw – ignore subsequent working
   - awrt – answers which round to
   - SC: special case
   - oe – or equivalent
   - dM – dependent method mark
   - dp decimal places
   - sf significant figures
   - * The answer is given on the paper – apply cso
   - d… Dependent mark

4. All A marks are 'correct answer only' (cao), unless shown, for example, as A1ft to indicate that previous wrong working is to be followed through.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
    - If all but one attempt is crossed out, mark the attempt which is NOT crossed out provided it is not cursory.
    - either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt

7. Ignore wrong working or incorrect statements following a correct answer unless the mark scheme indicates otherwise.

8. Mark question parts separately unless the mark scheme indicates otherwise. If question parts are unlabelled mark in the order presented.

<table>
<tr><th>Question Number</th><th>Scheme</th><th>Notes</th><th>Marks</th></tr>
<tr><td>1</td><td colspan="2">$\mathrm{f}(x)=\frac{2}{3}x^3-3x^2+5x+\mathrm{e}^{2x}(1-2x)$, $x=\frac{2x^2+5}{4\mathrm{e}^{2x}+6}$</td><td></td></tr>
<tr><td>(a)</td><td colspan="2">$\mathrm{f}'(x)=Ax^2\pm Bx+5\ \{+...\}\ A,B>0\ \ \{=2x^2-6x+5+...\}$
M1: Differentiates the first three terms to achieve $Ax^2\pm Bx+5\ \ A,B>0$
May be unsimplified but indices must be processed</td><td>M1</td></tr>
<tr><td></td><td colspan="2">$\mathrm{e}^{2x}(1-2x)\rightarrow 2\mathrm{e}^{2x}(1-2x)-2\mathrm{e}^{2x}$ or $\mathrm{e}^{2x}-2x\mathrm{e}^{2x}\rightarrow 2\mathrm{e}^{2x}-2\mathrm{e}^{2x}-4x\mathrm{e}^{2x}$
M1: Obtains one of the following forms for the product:
$\mathrm{e}^{2x}(1-2x)\rightarrow\pm C\mathrm{e}^{2x}(1-2x)\pm D\mathrm{e}^{2x}$
or $\mathrm{e}^{2x}-2x\mathrm{e}^{2x}\rightarrow\pm E\mathrm{e}^{2x}\pm F\mathrm{e}^{2x}\pm Gx\mathrm{e}^{2x}$
All coefficients non-zero. Condone if an incorrect rule is stated/used.
The product must produce all the appropriate terms so do not allow equivalents
e.g., $\mathrm{e}^{2x}-2x\mathrm{e}^{2x}\rightarrow 2\mathrm{e}^{2x}-4x\mathrm{e}^{2x}$ is M0
If only the $x\mathrm{e}^{2x}$ term is seen it must be the correct $-4x\mathrm{e}^{2x}$ from correct work for the product
A1: $(\mathrm{f}'(x)=)\ 2x^2-6x+5-4x\mathrm{e}^{2x}$
This answer only. Terms in any order. Not $5x^0$. May be partially factorised but Isw.</td><td>M1
A1</td></tr>
<tr><td colspan="4" align="right">(3)</td></tr>
<tr><td>(b)</td><td colspan="2">$\mathrm{f}'(0)=2(0)^2-6(0)+5-4(0)\mathrm{e}^{2(0)}=...\ \ \{5\}$ and
$\mathrm{f}'(1)=2(1)^2-6(1)+5-4(1)\mathrm{e}^{2(1)}=...\ \ \{-28.556...\}$
Obtains values for both $\mathrm{f}'(0)$ and $\mathrm{f}'(1)$ with at least one correct f '(1) = awrt -29 or -28. Allow ft on their f '(x) correct to 2 sf (allowing if truncated) provided it is of the form: $\pm Hx^2\pm Ix\pm J\pm$ at least one term of the form $\pm K\mathrm{e}^{2x}$ or $\pm Lx\mathrm{e}^{2x}$ oe with no other forms of terms but allow the J to be missing. No substitution needs to be seen.
M0 for clear use of f(x) instead of f '(x) but condone mislabelling i.e., sight of "f(0) = " and/or "f(1) =" provided there is no conclusive evidence that f(x) has actually been used.
Note that f(0) =1 (but note that this could be the same as their f '(0)) and f(1) = - 4.722...</td><td>M1</td></tr>
<tr><td></td><td>$\mathrm{f}'(0)=5$ and $\mathrm{f}'(1)=-28.556...$
$\Rightarrow$ There is a sign change and $\mathrm{f}'(x)$ is continuous over the interval hence root (in the interval $[0,1]$)</td><td>Both correct values from a correct f '(x), correct reason, mention of continuity and conclusion. An acceptable minimum in bold but allow equivalents e.g., "f '(0)>0, f '(1)<0" for "sign change" and condone e.g., "x/root/graph is continuous/continual". This is not a ft mark.</td><td>A1</td></tr>
<tr><td colspan="4" align="right">(2)</td></tr>
<tr><td>(c)(i)</td><td colspan="2">$\{x_1=0\Rightarrow x_2=\}\frac{5}{10}=\frac{1}{2}\Rightarrow x_3=\frac{2(0.5)^2+5}{4\mathrm{e}^{2(0.5)}+6}=...=0.3259620992$
M1: Substitutes $x=0$ into the iterative formula and then substitutes the answer into the formula and achieves a value. Condone slips if there is working but if just a value/values is/are given we must see awrt 0.326 (3sf)
A1: Awrt 0.3260 (4sf) and accept "0.326"
May just give answer only. Accept if unlabelled/mislabelled but mark their final answer to (i) in such cases (but isw if clearly followed by just rounding)</td><td>M1
A1</td></tr>
<tr><td>(c)(ii)</td><td colspan="2">$\{\alpha=\}\ 0.3675\ \ \{0.3674712656\}$</td><td>A1</td></tr>
</table>

	Awrt 0.3675. **Treat as a B mark** so 001 is a possible mark profile. Accept if unlabelled/mislabelled but mark their final answer to (ii) in such cases (but isw if followed by rounding)	
		(3)
		(Total 8 marks)

Question Number	Scheme	Notes	Marks
2	**"In this question you must show all stages of your working. Solutions relying entirely on calculator technology are not acceptable."** $\mathrm{f}(x)=3+4\mathrm{e}^{2x} \quad x\in\mathbb{R} \qquad \mathrm{g}(x)=\sqrt{x+5} \quad x\in\mathbb{R},\ x>-5$		
(a)	$\mathrm{f}(x)>3$	$\mathrm{f}(x)>3$ oe e.g., $(3,\ \infty)$ not $(3,\ \infty]$ Allow $y>3$, range > 3, f > 3, "> 3" but not e.g., $x>3$. No need for "$\mathbb{R}$"	**B1**
			(1)
(b)	$\mathrm{g}^{-1}(x)=x^2-5$ $x>0$ oe	M1: Swaps letters and rearranges to find the inverse (either order) and achieves the form $\{y=\}\pm x^2\pm 5$ A1: $\{\mathrm{g}^{-1}(x)=\}x^2-5$ Ignore labelling B1: x^2-5 **and** $x>0$ oe e.g., $0<x<\infty$	**M1** **A1** **B1**
			(3)
(c)	$\mathrm{f}\left(\ln\sqrt{7}\right)=3+4\times 7=31$ $\Rightarrow \mathrm{gf}\left(\ln\sqrt{7}\right)=\mathrm{g}(31)=...,\ 6$ or $\mathrm{gf}(x)=\sqrt{3+4\mathrm{e}^{2x}+5}$ $\Rightarrow \mathrm{gf}\left(\ln\sqrt{7}\right)=\sqrt{3+4\times 7+5}=...,\ 6$	M1: Finds a value $\mathrm{gf}\left(\ln\sqrt{7}\right)$ via a credible method. Only condone slips if there is sufficient working that makes their intention clear. So do not accept just $\mathrm{f}\left(\ln\sqrt{7}\right)=\alpha,\ \mathrm{g}(\alpha)=\beta$ or $\mathrm{gf}\left(\ln\sqrt{7}\right)=\beta$ unless it is correct A1: 6 (not ± 6) Accept answer only.	**M1** **A1**
			(2)
(d)	$3+4\mathrm{e}^{2a}=8$ B1: Sets $3+4\mathrm{e}^{2a}=8$ not just g(59) Condone $=\pm 8$. May use another letter for a provided it is not subsequently changed into a different expression. Do not award just for $3+4\mathrm{e}^{2\sqrt{a^2-5+5}}=8$ - the exponential part must be fully processed. May be scored later in their answer		**B1**
	$3+4\mathrm{e}^{2a}=8\Rightarrow \mathrm{e}^{2a}=...\ \left\{\frac{5}{4}\right\}\ \Rightarrow a=...,\ a=\ln\left(\frac{\sqrt{5}}{2}\right)$ M1: Makes e^{2a} the subject, from an equation of the form $3+4\mathrm{e}^{2a}=k\ (k>3)\Rightarrow 4\mathrm{e}^{2a}=c,\ c\neq k$ **and** takes lns of both sides (the step when lns are applied must be correct for their equation and note e.g., $\ln\left(\mathrm{e}^{2a}\right)$ or $2a\ln\mathrm{e}$ is not far enough) **and** proceeds to a real value for a (which may be decimal) so do not allow ln (negative) A1: $(a=)\ \ln\left(\frac{\sqrt{5}}{2}\right)$ or exact equivalent in the required "ln p" form e.g. $\ln\left(\sqrt{1.25}\right),\ \ln\left(\sqrt{\frac{5}{4}}\right),\ \ln\left(\frac{\sqrt{5}}{\sqrt{4}}\right)$ but not just e.g., $\ln\left(\frac{5}{4}\right)^{\frac{1}{2}}$ or $\ln\left(\left(\frac{5}{4}\right)^{\frac{1}{2}}\right)$ or $\frac{1}{2}\ln 1.25$ No other unrejected answers. Condone throughout if e.g., x is used for a		**M1** **A1**

(3)
(Total 9 marks)

Question Number	Scheme	Notes	Marks
3	**"In this question you must show all stages of your working. Solutions relying entirely on calculator technology are not acceptable."** $\mathrm{f}(x)=\dfrac{4x^2-14x+15}{x-3} \qquad x<3$		
(a)	$\mathrm{f}'(x)=\dfrac{(x-3)(8x-14)-(4x^2-14x+15)}{(x-3)^2}$ or e.g., $\dfrac{8x-14}{x-3}-\dfrac{4x^2-14x+15}{(x-3)^2}$ M1: Attempts the quotient rule to achieve the form $\dfrac{\pm(x-3)(\pm Ax\pm B)\pm(4x^2-14x+15)}{(x-3)^2}$ or $\dfrac{\pm(4x^2-14x+15)\pm(x-3)(\pm Ax\pm B)}{(x-3)^2}$ Alternatively, may use the product rule achieving the form $\pm(x-3)^{-2}\left(4x^2-14x+15\right)\pm(x-3)^{-1}\left(\pm Ax\pm B\right)$ Allow if there is a clear miscopy e.g. $15\to5$ Allow if an incorrect rule is used/stated provided correct form achieved. If $(x-3)(8x-14)$ is only seen expanded it must be correct and not $(x-3)"(8x-14)"$ but allow sign errors if a "-" is immediately applied to either $8x^2-38x+42$ or $4x^2-14x+15$. If the denominator is only seen expanded it must the correct x^2-6x+9 A1: Correct unsimplified derivative **seen**. Bracketing must be correct unless recovered.		**M1** **A1**
	$=\dfrac{(x-3)(8x-14)-(4x^2-14x+15)}{(x-3)^2}=\dfrac{8x^2-24x-14x+42-4x^2+14x-15}{(x-3)^2}=\dfrac{4x^2-24x+27}{(x-3)^2}$ **d**M1: Multiplies out the numerator and simplifies it to a three-term quadratic or equivalent work following use of the product rule achieving $\dfrac{ax^2+bx+c}{(x-3)^2}$ A1: Correct expression. Only allow values for a, b and c if the expression on the question paper is seen in their answer. Allow all marks if the denominator is expanded correctly but isw following the correct expression above		**dM1** **A1**
Alt by division first	$\dfrac{4x^2-14x+15}{x-3}=\dfrac{(4x-2)(x-3)+9}{x-3}=4x-2+9(x-3)^{-1}\Rightarrow\dfrac{\mathrm{d}y}{\mathrm{d}x}=4-9(x-3)^{-2}$ (M1:correct form, A1) $=4-\frac{9}{(x-3)^2}=\frac{4x^2-24x+27}{(x-3)^2}$ (**d**M1:correct form, A1)		
			(4)
(b)	$4x^2-24x+27\{=(2x-9)(2x-3)\}=0\Rightarrow x=\dfrac{3}{2},\ y=-2$ M1: Sets their 3TQ equal to zero and obtains a real value/expression for x. Apply usual rules but if calculator used they must get a correct real root for their 3TQ which may be $\geqslant 3$ A1: $\left(\frac{3}{2},-2\right)$ **from correct work and a correct derivative in (a)**. Allow coordinates given as $x=...,\ y=...$ No other unrejected solution offered.		**M1** **A1**
			(2)
(c)	$\left(\frac{5}{4},-7\right)$ and condone $\frac{5}{4},-7$ B1: One correct **d**B1: Both correct. Allow coordinates given as $x=...,\ y=...$ and accept any equivalents for $\frac{5}{4}$. Allow SC B1dB0 if just $\left(-7,\frac{5}{4}\right)$. B0**d**B0 for transforming the turning		**B1** **dB1**

	point "$\left(\frac{3}{2}, -2\right)$" B1B0 if there are other unrejected answers but ignore coordinates that are clearly just part of their solution such as $\left(\frac{5}{4}, -10\right)$ from applying f(2x)	
		(2)
		(Total 8 marks)

Question Number	Scheme	Notes	Marks
4	**"In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable."**		
	$(\sin 3x+\cos 3x)^2=\sin^2 3x+\cos^2 3x+2\sin 3x\cos 3x=1+2\sin 3x\cos 3x=1+\sin 6x$ Expands correctly and obtains an appropriate clearly integrable function: $(\sin 3x+\cos 3x)^2 \to \pm 1\pm\sin 6x$ (identity sign errors only). See variation below and Alt		**M1**
	$\int_{\frac{\pi}{18}}^{\frac{\pi}{12}} 1+\sin 6x \, \mathrm{d}x=\left[x-\frac{\cos 6x}{6}\right]_{\frac{\pi}{18}}^{\frac{\pi}{12}}$	**d**M1: Integrates $\pm 1\pm\sin 6x$ to $\pm x\pm r\cos 6x$ A1: $x-\frac{\cos 6x}{6}$ Must not be fortuitous. May have done this with more than one integral	**dM1** **A1**
	$\left(\frac{\pi}{12}-\frac{\cos\left(\frac{\pi}{2}\right)}{6}\right)-\left(\frac{\pi}{18}-\frac{\cos\left(\frac{\pi}{3}\right)}{6}\right)=\frac{\pi}{12}-\frac{\pi}{18}-0+\frac{1}{12}=\frac{\pi}{36}+\frac{1}{12}$ M1: Substitutes in the correct limits following an attempt at integration and subtracts either way round to obtain a two term expression of the form $a\pi+b,\ a,b\neq 0$ and b may be decimal. Allow slips if intention clear. If there is a substitution e.g., $u=6x$ and they do not revert to x then the changed limits must be correct e.g., $\frac{\pi}{2}$ and $\frac{\pi}{3}$ A1: Correct exact two term numerical expression. May be factorised (Isw) Must not be fortuitous.		**M1** **A1**
	An unsimplified variation on the above is to reach $\frac{1-\cos 6x}{2}+\frac{1+\cos 6x}{2}+\sin 6x$ (M1: identity sign errors only) $\Rightarrow \frac{1}{2}x-\frac{\sin 6x}{12}+\frac{1}{2}x+\frac{\sin 6x}{12}-\frac{\cos 6x}{6}$ (**d**M1A1) For the **d**M1 they must be integrating an expression that is equivalent to $\pm 1\pm\sin 6x$ and they must achieve an expression that is equivalent to $\pm x\pm r\cos 6x$. Final M1 and A1 as before noting that the limits may be applied to separate terms		
			(Total 5 marks)
Alt	$(\sin 3x+\cos 3x)^2=\sin^2 3x+\cos^2 3x+2\sin 3x\cos 3x=1+2\sin 3x\cos 3x$ $\int_{\frac{\pi}{18}}^{\frac{\pi}{12}} 1+2\sin 3x\cos 3x \, \mathrm{d}x=\left[x+\frac{\sin^2 3x}{3}\right]_{\frac{\pi}{18}}^{\frac{\pi}{12}}$ or $\left[x-\frac{\cos^2 3x}{3}\right]_{\frac{\pi}{18}}^{\frac{\pi}{12}}$ M1**d**M1: Obtains $\pm 1\pm 2\sin 3x\cos 3x$ and integrates to $\pm x\pm A\sin^2 3x$ or $\pm x\pm B\cos^2 3x$ The two M marks are awarded together here. Note that they could also use $\sin^2 3x+\cos^2 3x=\frac{1-\cos 6x}{2}+\frac{1+\cos 6x}{2}$ as in the main scheme variation A1: Correct integration		**M1** **dM1** **A1**
	$\left(\frac{\pi}{12}+\frac{\sin^2\left(\frac{\pi}{4}\right)}{3}\right)-\left(\frac{\pi}{18}+\frac{\sin^2\left(\frac{\pi}{6}\right)}{3}\right)$ or $\left(\frac{\pi}{12}-\frac{\cos^2\left(\frac{\pi}{4}\right)}{3}\right)-\left(\frac{\pi}{18}-\frac{\cos^2\left(\frac{\pi}{6}\right)}{3}\right)=\ldots.\quad \frac{\pi}{36}+\frac{1}{12}$ M1: Substitutes in the limits following an attempt at integration and subtracts either way round to reach $a\pi+b,\ a,b\in\mathbb{R}$. Trig evaluated. b may be irrational. Allow slips if intention clear.		**M1** **A1**

	A1: Correct exact two term numerical expression. May be factorised. Isw if necessary	
Note	If an alternative method of integration is used score the first M for reaching a clearly integrable function, **d**M1 for achieving a correct form after integration and A1 if completely correct. Allow sign errors only with identities used.	

Question Number	Scheme	Notes	Marks
5	$H = ab^t \Rightarrow \log_{10} H = \log_{10} a + (\log_{10} b)t$		
(a)	$\log_{10} a = 2.5$ **or** $\log_{10} b = \dfrac{3.25-2.5}{10-0}\ \left\{= \dfrac{3}{40},\ 0.075\right\}$	One correct equation in any form. Implied by appropriate expressions/values. The base of 10 may be implied later and allow e.g., lg *a*	**M1**
	$a = 10^{2.5}$ or $10^{\frac{5}{2}}$ or awrt 316 $b = 10^{\frac{3.25-2.5}{10-0}}$ or $10^{0.075}$ or $10^{\frac{3}{40}}$ or awrt 1.19	Any correct expression (or value) for *a* or *b*	**A1**
	$\log_{10} a = 2.5$ **and** $\log_{10} b = \dfrac{3.25-2.5}{10-0}\ \left\{= \dfrac{3}{40},\ 0.075\right\}$	Both correct equations in any form. Implied by appropriate expressions/values. The bases of 10 may be implied later	**dM1**
	Allow the above marks to be implied by the equation $H = 10^{\frac{5}{2}} \times 10^{\frac{3}{40}t}$ **but** $H = 10^{\frac{5}{2}+\frac{3}{40}t}$ or $H = 10^{2.5+0.075t}$ or $\log_{10} H = \dfrac{3}{40}t + 2.5$ need further work		
	$a = \text{awrt } 316,\ b = \text{awrt } 1.19$ Note: $a = 316.22776...,\ b = 1.18850...$	Both correct **decimal** values to **3sf** Answers only are acceptable. Accept values just embedded in the **given** model i.e., $H = 316 \times 1.19^t$ or e.g., $(1.19)^t.316$	**A1**
			(4)
(b)	e.g. {the number of households recycling their household waste} is **<u>increasing</u>** by **<u>19%</u>** **<u>per year</u>** **oe** ft their *b* from part (a) provided $b > 1$ and percentage correct to 2sf All 3 underlined parts are required but accept surrogates such as rising/going up....by a rate/factor/multiplier of "1.19"...year on year/each year/per annum Do not accept: The amount it is rising/The rate by which it increases/The percentage increase each year/The annual rate of increase/It is **changing** by 19% per year/The annual increase is 119%		**B1ft**
			(1)
(c)	*Solutions relying entirely on calculator technology are not acceptable.*		
	$\left\{\dfrac{\mathrm{d}H}{\mathrm{d}t}\right\} = 316 \times 1.19^t \times \ln(1.19),\ \Rightarrow \left\{\dfrac{\mathrm{d}H}{\mathrm{d}t}_{t=5} = \right\} \text{awrt } 130$ M1: Differentiates to the form $ab^t(\ln b)$oe - **correct** for their positive constants. Not e.g., "log" unless ln is later implied. Allow this mark with "*a*" and "*b*" instead of values. *t* = 5 may be already substituted and allow if it is partially calculated provided there is at least a product of two numbers seen (and those numbers are correct to 2sf) and it is correct for their values. A minimum would be e.g., 316×0.415 A1: Obtains awrt 130 (**2 sf**). Not ft Answer only of 130 (131.176059... or 129.50219... (accurate)) is 00		**M1** **A1**

(2)
(Total 7 marks)

Question Number	Scheme	Notes	Marks
6	$x=\left(3\sin 2y-\dfrac{6y}{\pi}\right)^2=9\sin^2 2y-\dfrac{36}{\pi}y\sin 2y+\dfrac{36y^2}{\pi^2}$ or $x^{\frac{1}{2}}=3\sin 2y-\dfrac{6y}{\pi}$		
	e.g., $\dfrac{\mathrm{d}x}{\mathrm{d}y}=2\left(3\sin 2y-\dfrac{6y}{\pi}\right)\left(6\cos 2y-\dfrac{6}{\pi}\right)$ or $\dfrac{\mathrm{d}x}{\mathrm{d}y}=36\sin 2y\cos 2y-\dfrac{36}{\pi}\sin 2y-\dfrac{72y}{\pi}\cos 2y+\dfrac{72y}{\pi^2}$ or $\dfrac{1}{2}x^{-\frac{1}{2}}\dfrac{\mathrm{d}x}{\mathrm{d}y}=6\cos 2y-\dfrac{6}{\pi}$ and may see these as $\dfrac{\mathrm{d}y}{\mathrm{d}x}=1/\ldots$ or $1=2\left(3\sin 2y-\dfrac{6y}{\pi}\right)\left(6\cos 2y\dfrac{\mathrm{d}y}{\mathrm{d}x}-\dfrac{6}{\pi}\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)$ $1=36\sin 2y\cos 2y\dfrac{\mathrm{d}y}{\mathrm{d}x}-\dfrac{36}{\pi}(\sin 2y+2y\cos 2y)\dfrac{\mathrm{d}y}{\mathrm{d}x}+\dfrac{72y}{\pi^2}\dfrac{\mathrm{d}y}{\mathrm{d}x}$ or $\dfrac{1}{2}x^{-\frac{1}{2}}=\left(6\cos 2y-\dfrac{6}{\pi}\right)\dfrac{\mathrm{d}y}{\mathrm{d}x}$ M1: Differentiates to achieve any correct form (sign/coefficient errors only) of an equation in $\dfrac{\mathrm{d}x}{\mathrm{d}y}\left(\text{or }\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)$. Bracketing errors must be recovered. If any trig identities are used e.g., $\sin 2y\cos 2y\rightarrow\frac{1}{2}\sin 4y$ condone sign errors only but ISW. Note that $\frac{\mathrm{d}}{\mathrm{d}y}\left(9\sin^2 2y\right)=\frac{\mathrm{d}}{\mathrm{d}y}\left(\frac{9}{2}-\frac{9}{2}\cos 4y\right)=18\sin 4y$ A1: Correct differentiation. ISW once a correct equation is seen		M1 A1
	$\dfrac{\mathrm{d}x}{\mathrm{d}y}=2\left(6\cos\dfrac{\pi}{2}-\dfrac{6}{\pi}\right)\left(3\sin\dfrac{\pi}{2}-\dfrac{3}{2}\right)=-\dfrac{18}{\pi}\ \{=-5.7\ldots\}\Rightarrow\dfrac{\mathrm{d}y}{\mathrm{d}x}=-\dfrac{\pi}{18}\ (=-0.17\ldots)$ Via $\frac{\mathrm{d}x}{\mathrm{d}y}$: Substitutes in $y=\frac{\pi}{4}$ (and $x=$"$\frac{9}{4}$" if necessary) and uses $\frac{\mathrm{d}y}{\mathrm{d}x}=1/\frac{\mathrm{d}x}{\mathrm{d}y}$ to find a value/numerical expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ with all trig evaluated, which could be a decimal. Implied by $\dfrac{\mathrm{d}y}{\mathrm{d}x}=-\dfrac{\pi}{18}$ $(=-0.17\ldots)$. This could be done in either order. There may be slips including incorrect reciprocation e.g., $1/\left(\frac{a}{b}+\frac{c}{d}\right)\rightarrow\frac{b}{a}+\frac{d}{c}$ but there must be evidence of an attempt to use the correct $\frac{\mathrm{d}y}{\mathrm{d}x}=1/\frac{\mathrm{d}x}{\mathrm{d}y}$ with the correct coordinate value/s and $\frac{\mathrm{d}y}{\mathrm{d}x}=-1/\frac{\mathrm{d}x}{\mathrm{d}y}$ is **d**M0. If only a value is given for $\frac{\mathrm{d}x}{\mathrm{d}y}$ followed by one for $\frac{\mathrm{d}y}{\mathrm{d}x}$ the values must imply $\frac{\mathrm{d}y}{\mathrm{d}x}=1/\frac{\mathrm{d}x}{\mathrm{d}y}$ Via $\frac{\mathrm{d}y}{\mathrm{d}x}$: Alternatively obtains a value/num. expression with all trig evaluated by substituting the correct values and making $\frac{\mathrm{d}y}{\mathrm{d}x}$ the subject (either order). Allow slips.		dM1
	x coordinate of P is $\dfrac{9}{4}$ or $\{P\}\left(\dfrac{9}{4},\ \dfrac{\pi}{4}\right)$	Correct x-coordinate of P Allow equivalent single fractions. No working required	B1
	$0-\dfrac{\pi}{4}=-\dfrac{\pi}{18}\left(x-\dfrac{9}{4}\right)\Rightarrow x=\ldots,\ \dfrac{27}{4}$ or $\dfrac{\pi}{4}=-\dfrac{\pi}{18}\left(\dfrac{9}{4}\right)+c\Rightarrow c=\dfrac{3\pi}{8}\Rightarrow-\dfrac{\pi}{18}x=-\dfrac{3\pi}{8}\Rightarrow x=\ldots,\ \dfrac{27}{4}$ M1: Uses a straight line equation method - initial equation correct- ft their value/expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ with their non-zero x coordinate and the given y coordinate and proceeds to find a value/expression for x by setting $y=0$. Not a dependent mark but must follow a credible attempt to differentiate and obtain a value for $\frac{\mathrm{d}y}{\mathrm{d}x}$ using $y=\frac{\pi}{4}$ appropriately and do not condone clear use of an unchanged $\frac{\mathrm{d}x}{\mathrm{d}y}$ in the line equation. A1: Correct x-coordinate of Q. Requires all previous marks.		M1 A1

(Total 6 marks)

Question Number	Scheme	Marks
7	**"In this question you must show all stages of your working.** **Solutions relying entirely on calculator technology are not acceptable."** $P_C = 7+16e^{-0.1t}$ $\qquad P_V = 4+me^{-0.2t}$ $\qquad 0 \leqslant t \leqslant 15$	
(a)	$7+16e^{-0.1T} = 15 \Rightarrow e^{-0.1T} = ...$ Sets $7+16e^{-0.1T} = 15$ and obtains $e^{-0.1T} = k$	**M1**
	$e^{-0.1t} = \dfrac{8}{16} \Rightarrow T = \dfrac{\ln(0.5)}{-0.1} = \text{awrt } 6.93$ **d**M1: Finds a positive value for T - allow slips & $T \geqslant 15$ but must be correct ln work A1: awrt 6.93 May use t or x	**dM1** **A1**
		(3)
(b)	$\dfrac{dP_C}{dt} = -1.6e^{-0.1t}$ or $-\dfrac{8}{5}e^{-0.1t} \Rightarrow -1.6e^{-0.1\times 5} = \text{awrt } -0.97$ M1: Differentiates to the form $Ae^{-0.1t}$, $A \neq 16$, substitutes in $t=5$ and obtains a decimal value. There must be some indication of an appropriate method but accept the derivative (with or without substitution) followed by a value. A1: awrt -0.97 **Answer only is M0A0**	**M1** **A1**
		(2)
(c)(i)	$\{m=\}\ 32$ Correct value. Ignore any labelling. Not 32000 and not both 32 and 32000 unless one is rejected	**B1**
(ii)	$36(000)-23(000)=13(000)$ M1: "36"-23 or $36000-23000$ either way round with their positive "36" = m + 4 or "36000" = m + 4000 as necessary (no inconsistent units in the subtraction) A1: $\{£\}\pm 13000$ or 13 thousand (not 13 on its own or with 13000)	**M1** **A1**
		(3)
(d)	$4+\text{"}32\text{"}e^{-0.2\alpha} = 7+16e^{-0.1\alpha} \Rightarrow 32e^{-0.2\alpha}-16e^{-0.1\alpha}-3=0$ or e.g., $3e^{0.2\alpha}+16e^{0.1\alpha}-32=0$ Sets $4+\text{"}32\text{"}e^{-0.2\alpha} = 7+16e^{-0.1\alpha}$ and obtains a 3TQ in $e^{\pm 0.1\alpha}$ (terms may not be on one side). May use any variable or a substitution e.g. $\beta = e^{-0.1\alpha} \Rightarrow 32\beta^2-16\beta-3=0$ The 3TQ and the "=" could be implied by an attempt to solve a 3TQ in $e^{-0.1\alpha}$ or $e^{+0.1\alpha}$	**M1**
	$\Rightarrow e^{-0.1\alpha} = \dfrac{16\pm\sqrt{16^2-4(32)(-3)}}{2(32)}\left\{\Rightarrow \dfrac{2+\sqrt{10}}{8}\right\}$ or $e^{0.1\alpha} = \dfrac{-16\pm\sqrt{16^2-4(3)(-32)}}{2(3)}\left\{\Rightarrow \dfrac{-8+4\sqrt{10}}{3}\right\}$ Solves their three term quadratic in $e^{\pm 0.1\alpha}$ or "β" (usual rules apply but must get a real answer (could be negative) via formula or CTS and not factorisation). Allow going straight to a consistent calculator answer provided there has been some (minimal) algebra. If decimal it must be correct to 2sf	**dM1**
	$e^{-0.1\alpha} = \dfrac{2+\sqrt{10}}{8} \Rightarrow \{\alpha=\}\dfrac{\ln\left(\dfrac{2+\sqrt{10}}{8}\right)}{-0.1} = \text{awrt } 4.38$ or $e^{0.1\alpha} = \dfrac{-8+4\sqrt{10}}{3} \Rightarrow \{\alpha=\}\dfrac{\ln\left(\dfrac{-8+4\sqrt{10}}{3}\right)}{0.1} = \text{awrt } 4.38$ **dd**M1: Obtains a valid positive decimal value (not just changed from a negative) - may be slips but must involve correct application of lns. Accept 4.38 following a value for $e^{-0.1\alpha}$ or $e^{+0.1\alpha}$ A1: awrt 4.38. Allow if unlabelled or mislabelled e.g., "a", "x", "t"	**ddM1** **A1**
	Minimum work: $4+\text{"}32\text{"}e^{-0.2\alpha} = 7+16e^{-0.1\alpha}$, $\quad e^{-0.1\alpha} = \dfrac{2+\sqrt{10}}{8}$, $\quad \alpha = 4.3806365...$ 4.38 without the root and an equation will score no marks.	

(4)
(Total 12 marks)

<table>
<tr><th>Question Number</th><th colspan="2">Scheme</th><th>Marks</th></tr>
<tr><td>8</td><td colspan="2">"In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable."</td><td></td></tr>
<tr><td>(a)</td><td colspan="2">$$\frac{\sec\theta\operatorname{cosec}\theta}{4\operatorname{cosec}4\theta}\left\{=\frac{\sin4\theta}{4\cos\theta\sin\theta}\right\}=\frac{2\sin2\theta\cos2\theta}{2\sin2\theta}=\cos2\theta\,*$$
M1: Clearly uses either $\sin2\theta=2\sin\theta\cos\theta$ or $\sin4\theta=2\sin2\theta\cos2\theta$ or equivalent double angle work in the LHS (or RHS) in a correct step with no errors beforehand
A1*: Uses both $\sin2\theta=2\sin\theta\cos\theta$ and $\sin4\theta=2\sin2\theta\cos2\theta$ or equivalent work and obtains the given answer with no errors. Only one intermediate stage of working is required by the route above and condone starting with the expression in brackets above. No missing/mixed variables at any stage. Attempts that start on the RHS mark the same way and may start with e.g. $\frac{2\sin2\theta\cos2\theta}{2\sin2\theta}$ or $\frac{\sin2\theta\cos2\theta}{\sin2\theta}$.
Allow meet in the middle proofs. "1=1" proofs need a (minimal) conclusion such as "shown".</td><td>M1
A1*</td></tr>
<tr><td></td><td colspan="2"></td><td>(2)</td></tr>
<tr><td>(b)</td><td colspan="2">$$4\left(5\cos^2\theta-9\right)\operatorname{cosec}4\theta=3\left(1+\tan^2 2\theta\right)\sec\theta\operatorname{cosec}\theta \qquad 0<x<180^\circ$$</td><td></td></tr>
<tr><td></td><td>e.g., $\frac{5\cos^2\theta-9}{3\left(1+\tan^2 2\theta\right)}=\cos2\theta$ or
$5\cos^2\theta-9=3\left(1+\frac{\sin^2 2\theta}{\cos^2 2\theta}\right)\cos2\theta$ or
$5\cos^2\theta-9=3(\sec^2 2\theta)\cos2\theta$</td><td>Correct use of part (a) to achieve $\frac{5\cos^2\theta-9}{3\left(1+\tan^2 2\theta\right)}=\cos2\theta$ or better.
Must see a correct equation.
Could repeat the work in part (a)</td><td>B1</td></tr>
<tr><td></td><td colspan="2">$$5\frac{(\cos2\theta+1)}{2}-9=3\sec^2 2\theta\cos2\theta\Rightarrow5\cos2\theta+5-18=\frac{6}{\cos2\theta}\Rightarrow5\cos^2 2\theta-13\cos2\theta-6\{=0\}$$
M1: Uses both $\cos2\theta=2\cos^2\theta-1$ and $1+\tan^2 2\theta=\sec^2 2\theta$ or sufficient equivalent identities in their equation and obtains a 3TQ in $\cos2\theta$. Allow slips in the manipulation but errors with identities can only be sign errors. If they repeat the work in part (a) it must be done correctly.
A1: Correct 3TQ. Condone a missing "=0" and recovery including bracketing issues, missing/mixed variables and poor notation but must not be fortuitous. Allow any multiple.</td><td>M1
A1</td></tr>
<tr><td></td><td colspan="2"></td><td>(3)</td></tr>
<tr><td>(c)</td><td colspan="2">Allow invented non-zero values for a, b and c for the first two marks only.
$$\left(5\cos2x+2\right)\left(\cos2x-3\right)=0\Rightarrow\left(\cos2x=\right)-\tfrac{2}{5}$$
Solves their 3TQ in $\cos2x$ or $\cos2\theta$ from (b) to obtain $\cos2x=k$ where $|k|<1$
Usual rules but if calculator used must get one correct root between -1 and 1 for their equation.
May have set e.g., $y=\cos2x$ but do not accept e.g. $x=-0.4$ unless recovered</td><td>M1</td></tr>
<tr><td></td><td colspan="2">$$\left(\cos2x=\right)-\frac{2}{5}\Rightarrow\{x=\}...$$
Obtains any correct 2sf ft value for x in degrees or radians (you may need to check) from their $\cos2x="-\frac{2}{5}"$ $\left\{\text{Note}: -\tfrac{2}{5}\Rightarrow0.991 \text{ radians (awrt } 0.99)\right\}$</td><td>dM1</td></tr>
<tr><td></td><td colspan="2">$$\{x=\}\ \ 56.789...,\ 123.21...\ \Rightarrow \text{awrt } 56.8,\ \text{awrt } 123.2$$
A1: One of awrt 57 or 123 (may be others in the $0<x<180^\circ$ range)</td><td>A1
A1</td></tr>
</table>

	A1: Both awrt 56.8 and 123.2 and no others in the range or multiples of 45. No degrees symbol required. Ignore labelling. A marks require a correct 3TQ & correct roots seen. Answers only is 0/4	
		(4)
		(Total 9 marks)

Question Number	Scheme	Notes	Marks
9	**"In this question you must show all stages of your working. Solutions relying entirely on calculator technology are not acceptable."**		
(a)	$\int \frac{11}{2x+5}+4\mathrm{e}^{x-3}\ \mathrm{d}x = \frac{11}{2}\ln(2x+5)+4\mathrm{e}^{x-3}\quad (+\,c)$ M1: Integrates to the form $\pm A\ln(2x+5)\pm B\mathrm{e}^{x-3}\quad A,B\neq 0$ Terms may be seen separately. A1: Correct integration. Terms together. Constant not required. May be unsimplified Condone missing brackets i.e., $\frac{11}{2}\ln 2x+5+4\mathrm{e}^{x-3}$		**M1** **A1**
			(2)
(b)	$y=a-\lvert bx-1\rvert \Rightarrow \left(\frac{1}{b},\ a\right)$ B1: Either correct coordinate **d**B1: Both correct coordinates Accept coordinates as "$x=...,\ y=...$" Allow SC B1dB0 if just $\left(a,\ \frac{1}{b}\right)$		**B1** **dB1**
			(2)
(c)	$(0,\ 9)\Rightarrow 9=a-1\Rightarrow a=10$	Correct value for a. No other values	**B1**
	$x=3\Rightarrow\{y=\}\frac{11}{2\times3+5}+4\mathrm{e}^{3-3}\quad\{=5\}$	Any correct expression or value seen or implied for the y-coordinate of intersection point. May be on diagram. Implied by $b=2$ if no clearly incorrect work	**M1**
	$5=10-3b+1\Rightarrow b=2$	Correct value for b. No other values	**A1**
			(3)
(d)	If (a) was omitted allow marks for (a) to be scored here. If reattempted, an answer to (a) takes priority for the marks for (a) but allow access to all the marks in (d)		
	$\left(\frac{11}{2}\ln(2\times3+5)+4\mathrm{e}^{3-3}\right)-\left(\frac{11}{2}\ln(5)+4\mathrm{e}^{-3}\right)\ \left\{=\frac{11}{2}\ln\frac{11}{5}+4-\frac{4}{\mathrm{e}^{3}}\right\}$ or $17.1888424...-9.05105...\ \{=8.137367\}..$ Substitutes in 3 and 0, subtracts the **right** way round and obtains a numerical expression or positive difference of two values. Minimum working is a subtraction of two values, unless the correct 8.137... is seen along with the correct integrated function (which may just be in (a)). However, this can also be implied by a final answer of awrt 14.4 provided there is no wrong working and the correct integrated function is seen (which may just be in (a)). Allow slips but must be a recognisable attempt at the f(3) - f(0) with $\mathrm{f}(0)\neq 0$. Requires an integrated function of the form $...\ln(\mathrm{f}(x))\pm...\mathrm{e}^{\mathrm{g}(x)}$. If restated/redone in (d) the work in (d) takes precedence. This work may be seen as part of a correct or incorrect overall method.		**M1**
	Crosses the x-axis at $\frac{11}{2}$	Correct x-intercept seen (may be on diagram) or implied Note that work in the main answer space takes precedence over work on the diagram	**B1**
	$\frac{1}{2}\times"5"\times\left("\frac{11}{2}"-3\right)+"8.137..."\left\{=\frac{25}{4}+"8.137..."\right\}=...,\quad 14.387867...\approx \text{awrt}\,14.4$ **d**M1: A full attempt to find the shaded area, i.e., adds any correct ft numerical expression (or value) for the triangle area with their y-coordinate of the intersecction and their x-coordinate of the x-axis intercept to their positive (not a corrected		**dM1** **A1**

	negative) area under the curve and obtains a value, provided 0 < their "5" < 9 and their $"\frac{11}{2}" > 3$. A1: awrt 14.4 **See note overleaf if integration is used for the triangle area**.	
		(4)
		(Total 11 marks)

9(d) **Note:** **integration for triangle**	Note: If integration of $y = a - \lvert bx - 1 \rvert \left(= a - bx + 1 \text{ for } x \geqslant 1\right)$ is used for the triangle they must obtain a numerical expression or value that is correct for their values and appropriate to the information given in the question. This requires: $\int_3^{"\frac{11}{2}"} ("10" - "2"x + 1)\,dx$ where $"\frac{11}{2}" > 3$, "10" (their a) $\geqslant 10$ & "2" (their b) > 0 and $a, b \in \mathbb{Z}$ $\left[-x^2 + 11x\right]_3^{"\frac{11}{2}"} = -\frac{121}{4} + \frac{121}{2} - (-9 + 33) = \frac{121}{4} - 24 = \frac{25}{4}$ $\Rightarrow "\frac{25}{4}" + "8.137..." = ..., \quad 14.387867... \approx \text{awrt } 14.4$	
	If points are used to find the equation of the line they must be in appropriate relative positions so they do not contradict information given in the question and implied by the diagram. So if using the maximum $\left("\frac{1}{2}", "10"\right)$, $0 < "\frac{1}{2}" < 3$ and "10" > 9 If using the intersection $(3, "5")$ then $3 < "5" < "\frac{11}{2}"$ If using the x-axis intercept $\left("\frac{11}{2}", 0\right)$ then $"\frac{11}{2}" > 3$ The work must be consistent with their a and b e.g., $y - "5" = \frac{"5" - 0}{"\frac{11}{2}" - 3}(x - 3) \Rightarrow y = -2x + 11 \Rightarrow \left[-x^2 + 11x\right]_3^{"\frac{11}{2}"} = -\frac{121}{4} + \frac{121}{2} - (-9 + 33) = \frac{121}{4} - 24 = \frac{25}{4}$ $\Rightarrow "\frac{25}{4}" + "8.137..." = ..., \quad 14.387867... \approx \text{awrt } 14.4$	