# Mark Scheme (Results)

# Summer 2026

Pearson Edexcel International GCSE

In Further Pure Mathematics (4PM1)

Paper 1R

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## General Marking Guidance

All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.

Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.

Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.

There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.

All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.

Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.

When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.

Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

- **Types of mark**
    - M marks: method marks
    - A marks: accuracy marks – can only be awarded when relevant M marks have been gained
    - B marks: unconditional accuracy marks (independent of M marks)

- **Abbreviations**
    - cao – correct answer only
    - cso – correct solution only
    - ft – follow through
    - isw – ignore subsequent working
    - SC - special case
    - oe – or equivalent (and appropriate)
    - dep – dependent
    - indep – independent
    - awrt – answer which rounds to
    - eeoo – each error or omission

- **No working**

  If no working is shown then correct answers may score full marks

  If no working is shown then incorrect (even though nearly correct) answers score no marks.

- **With working**

  If it is clear from the working that the "correct" answer has been obtained from incorrect working, award 0 marks.

  If a candidate misreads a number from the question: eg. uses 252 instead of 255; follow through their working and deduct 2A marks from any gained provided the work has not been simplified. (Do not deduct any M marks gained.)

  If there is a choice of methods shown, then award the lowest mark, unless the subsequent working makes clear the method that has been used

  Examiners should send any instance of a suspected misread to review (but see above for simple misreads).

- **Ignoring subsequent work**

  It is appropriate to ignore subsequent work when the additional work does not change the answer in a way that is inappropriate for the question: eg. incorrect cancelling of a fraction that would otherwise be correct.

It is not appropriate to ignore subsequent work when the additional work essentially makes the answer incorrect eg algebra.

- **Parts of questions**
  Unless allowed by the mark scheme, the marks allocated to one part of the question CANNOT be awarded to another.

## General Principles for Further Pure Mathematics Marking

(but note that specific mark schemes may sometimes override these general principles)

**Method mark for solving a 3 term quadratic equation:**

1. Factorisation:

$(x^2+bx+c)=(x+p)(x+q)$, where $|pq|=|c|$ leading to $x=....$

$(ax^2+bx+c)=(mx+p)(nx+q)$ where $|pq|=|c|$ and $|mn|=|a|$ leading to $x=....$

2. Formula:
Attempt to use the **correct** formula (shown explicitly or implied by working) with values for *a*, *b* and *c*, leading to $x=....$

3. Completing the square:

$x^2+bx+c=0$: $\left(x\pm\frac{b}{2}\right)^2\pm q\pm c=0,\quad q\neq 0$ leading to $x=....$

**Method marks for differentiation and integration:**

1. Differentiation

Power of at least one term decreased by 1. $\left(x^n\to x^{n-1}\right)$

2. Integration:

Power of at least one term increased by 1. $\left(x^n\to x^{n+1}\right)$

**Use of a formula:**

Generally, the method mark is gained by **either**

quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values

**or**, where the formula is not quoted, the method mark can be gained by implication from the substitution of correct values and then proceeding to a solution.

**Answers without working:**

The rubric states "Without sufficient working, correct answers may be awarded no marks".

General policy is that if it could be done "in your head" detailed working would not be required. (Mark schemes may override this eg in a case of "prove or show...."

**Exact answers:**

When a question demands an exact answer, all the working must also be exact. Once a candidate loses exactness by resorting to decimals the exactness cannot be regained.

**Rounding answers (where accuracy is specified in the question)**

Penalise only once per question for failing to round as instructed - ie giving more digits in the answers. Answers with fewer digits are automatically incorrect, but the is rule may allow the mark to be awarded before the final answer is given.

| Question number | Scheme | Marks |
|---|---|---|
| 1 | $31.85=\dfrac{1.3}{2}r^2 \Rightarrow r=\sqrt{49}=7$ (cm) | M1 |
| | $P=7+7+7\times 1.3=23.1$ cm | M1A1 [3] |
| | **Total** | **3 marks** |

| Mark | Notes |
|---|---|
| M1 | Uses the correct formula for the area of a sector correctly to find the radius. |
| M1 | Uses their $r$ in a complete method to find the perimeter using the correct formula for the length of arc. |
| A1 | For perimeter = 23.1 (This is an exact value) |

| Question number | Scheme $\Rightarrow$ | Marks |
|---|---|---|
| 2(a) | $S_n = \frac{n}{2}(3+5n) \Rightarrow S_1 = 4 \quad S_2 = 13$ | M1A1 |
| | $\Rightarrow a = 4 \quad a+a+d = 13 \Rightarrow d = 5$ | M1A1 |
| | $S_n = \sum_{r=1}^{n}\left(a+(r-1)d\right) = \sum_{r=1}^{n}\left(4+(r-1)5\right) \Rightarrow S_n = \sum_{r=1}^{n}(5r-1)$ * | M1A1 cso |
| | ALT (for the first 4 marks) | |
| | $\frac{n}{2}(3+5n) = \frac{n}{2}\left(2a+(n-1)d\right) \Rightarrow 3+5n = 2a+(n-1)d$ | M1 A1 |
| | $5n = dn \Rightarrow d = 5$ and $3 = 2a-5 \Rightarrow a = 4$ | M1 A1 [6] |
| (b) | $S_{n+3} = 4t_{20} - 18$ | |
| | $\frac{n+3}{2}\left(3+5(n+3)\right) = 4\times\left(4+(20-1)5\right) - 18 = 378$ | M1 |
| | $\Rightarrow (n+3)(18+5n) = 756 \Rightarrow 5n^2+33n-702 = 0$ | M1 |
| | $\Rightarrow (n-9)(5n+78) = 0 \Rightarrow n = ...$ | M1 |
| | $\Rightarrow n = 9, \quad \left[ n = -\frac{78}{5} \text{ need not be seen} \right]$ | A1 [4] |
| | **Total** | **10 marks** |

| Part | Mark | Notes |
|---|---|---|
| (a) | | **We are not allowing working backwards in part (a) from the given answer. If there is a mixture of methods, score only for the methods seen here below.** |
| | M1 | **This is a B mark in epen** Finds $S_1$ or $S_2$ |
| | A1 | **This is a B mark in epen** Finds $S_1$ and $S_2$ |
| | M1 | Uses their values to find a value for *a* and *d* |
| | A1 | For both *a*, the first term, and *d*, the common difference, correct |
| | | **ALT** for first **4** marks |
| | M1 | **This is a B mark in epen** Compares the given formula for *Sn* with the known formula for the sum of an arithmetic sequence |
| | A1 | **This is a B mark in epen** Eliminates $\frac{n}{2}$ to form the equation in *a*, *n* and *d* |
| | M1 | Equates coefficients to find a value for *d* and *a* |
| | A1 | For both *a*, the first term, and *d*, the common difference, correct |
| | M1 | Use the formula for the *n*th term $(a + (n-1)d)$ using their *a* and *d* to form the required expression. |
| | A1*cso | For the required expression with no errors |
| (b) | M1 | Uses their values of *a* and *d* to set up the expression |
| | M1 | Forms a 3TQ |
| | M1 | Solves their 3TQ by any method including calculator. If there is no explicit method shown, only award this mark for either correct value of *n* following a correct 3TQ |
| | A1 | For $n = 9$, and rejects the other value. This can be implied. |

| Question number | Scheme | Marks |
|---|---|---|
| 3<br><br>(a)(i) | Mark (a)(i) and (ii) together<br>$\alpha+\beta=\frac{5}{3}, \quad \alpha\beta=\frac{1}{3}$<br>$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta \Rightarrow \left(\frac{5}{3}\right)^2-2\left(\frac{1}{3}\right)=\frac{19}{9}$ | B1<br><br>M1A1<br>[3] |
| (ii)<br><br>ALT | $\alpha-\beta=\sqrt{(\alpha-\beta)^2}=\sqrt{(\alpha^2+\beta^2-2\alpha\beta)}=\sqrt{\left(\frac{19}{9}-2\left(\frac{1}{3}\right)\right)}=\frac{\sqrt{13}}{3}$ *<br><br>$\alpha-\beta=\sqrt{(\alpha-\beta)^2}=\sqrt{((\alpha+\beta)^2-4\alpha\beta)}$ also leads to the same value | M1A1<br>cso<br>[2] |
| (b) | Sum $\quad \frac{\alpha+\beta}{\alpha}+\frac{\alpha-\beta}{\beta}=\frac{\alpha\beta+\beta^2+\alpha^2-\alpha\beta}{\alpha\beta}=\frac{\frac{19}{9}}{\frac{1}{3}}=\frac{19}{3}$<br><br>Product $\quad \left(\frac{\alpha+\beta}{\alpha}\right)\times\left(\frac{\alpha-\beta}{\beta}\right)=\frac{\frac{5}{3}\times\frac{\sqrt{13}}{3}}{\frac{1}{3}}=\frac{5\sqrt{13}}{3}$<br><br>Equation $\quad x^2-\left(\frac{19}{3}\right)x+\left(\frac{5\sqrt{13}}{3}\right)=0 \Rightarrow 3x^2-19x+5\sqrt{13}=0$ oe | M1A1<br><br>M1A1<br><br>M1A1<br>[6] |
| | **Total** | **11 marks** |

| Part | Mark | Notes |
|---|---|---|
| (a)(i) | B1 | For the correct values for the sum and product |
| | M1 | For the correct algebra and substitution of their values for the sum and product to find a value for $\alpha^2+\beta^2$ |
| | A1 | For the correct value. |
| (ii) | M1 | For the correct algebra and substition to find a value for $\alpha-\beta$ |
| | A1* | For the correct value |
| (b) | M1 | For the correct algebra such that either $\alpha+\beta$ or $\alpha\beta$ or $\alpha^2+\beta^2$ can be substituted directly to find the SUM |
| | A1 | For the correct sum |
| | M1 | For the correct algebra such that either $\alpha+\beta, \alpha-\beta$ and $\alpha\beta$ can be substituted directly to find the PRODUCT |
| | A1 | For the correct product |
| | M1 | For a correct equation using their values |
| | A1 | For the correct equation [including = 0] with integer values. |

| Question number | Scheme | Marks |
|---|---|---|
| 4 (a) | $y = x(3x-4)^{\frac{1}{2}} \Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{1}{2}\times 3\times x\times(3x-4)^{-\frac{1}{2}} + (3x-4)^{\frac{1}{2}}\times 1$<br>$\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{3x+2(3x-4)}{2\sqrt{3x-4}} = \dfrac{9x-8}{2\sqrt{3x-4}}$ * | M1A1A1<br>A1<br>cso<br>[4] |
| (b) | $x = \dfrac{8}{3} \Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{9\times\frac{8}{3}-8}{2\times\sqrt{3\times\frac{8}{3}-4}} = \dfrac{16}{2\times 2} = 4$ | M1A1<br>[2] |
| (c) | $y = \dfrac{8}{3}\times\sqrt{3\times\dfrac{8}{3}-4} = \dfrac{16}{3}$<br>$\left(y-\dfrac{16}{3}\right) = \text{'}4\text{'}\left(x-\dfrac{8}{3}\right), \Rightarrow 3y-12x+16=0$ oe | B1<br>M1A1,A1<br>[4] |
| | **Total** | **10 marks** |

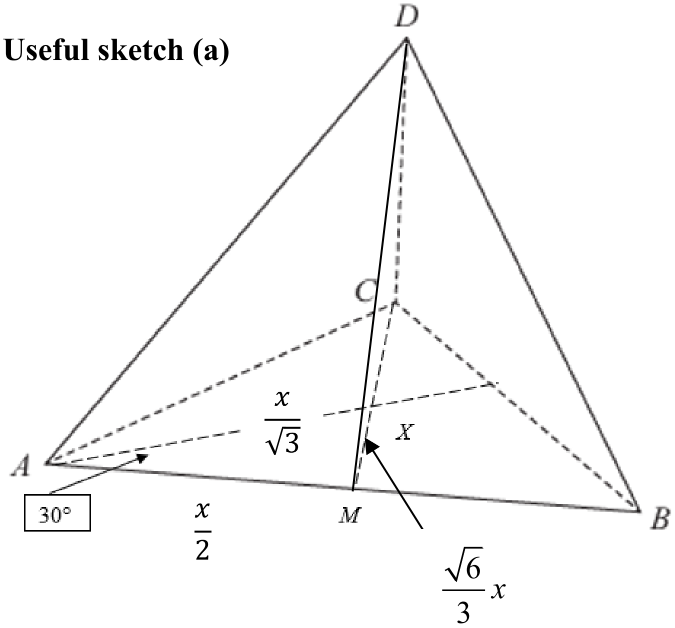
| Part | Mark | Notes |
|---|---|---|
| **Part** | **Mark** | **Notes** |
| (a) | M1 | Attempts to apply correct product rule correctly.<br>A minimally acceptable attempt is as follows:<br>• The structure must be correct with both terms added.<br>The minimum acceptable form is:<br>$\dfrac{\mathrm{d}y}{\mathrm{d}x} = x\times\alpha(3x-4)^{-\frac{1}{2}} + 1\times(3x-4)^{\frac{1}{2}}$ where $\alpha \neq 0, 1$ |
| | A1 | One term fully correct |
| | A1 | Completely correct. |
| (b)<br>(b) | A1*<br>cso | Rearranged to the form required with no omissions or errors seen. |
| | M1 | For the correct substitution of $x$ into the gradient formula which must be as a minimum $\dfrac{Ax}{2\sqrt{3x-4}}$ |
| (c)<br>(c) | A1 | For the correct value of the gradient of the tangent. If the gradient of the normal is used in part c) withold the mark in part c) only. |
| | B1 | For finding the correct value for $y$ |
| | M1 | For forming any equation of a straight line using their values and the value for the gradient. Use of the gradient of the normal is M0.<br>If they use $y = mx + c$ they must find $c$ **and** form an equation for the award of this mark. |
| | A1 | For a correct equation in any form |
| | A1 | For the correct equation in the required form. |

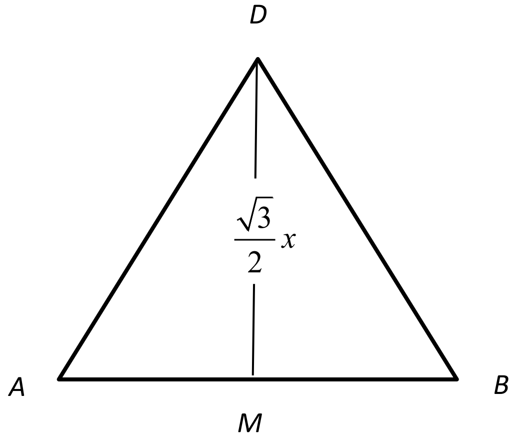
| Question number | Scheme | Marks |
|---|---|---|
| 5 (a) | $1 = 2\cos 3t \Rightarrow \cos 3t = \frac{1}{2} \Rightarrow 3t = \frac{\pi}{3} \Rightarrow t = \frac{\pi}{9}$ | M1A1<br>[2] |
| (b) | $a = \frac{\mathrm{d}v}{\mathrm{d}t} = -6\sin 3t \Rightarrow a = -6\sin\left(3\times\frac{\pi}{2}\right) = 6\ \left(\mathrm{m/s^2}\right)$ | M1dM1A1<br>[3] |
| (c) | $s = \int 2\cos 3t\ \mathrm{d}t = \frac{2}{3}\sin 3t + c$ | M1 |
| | $t = \frac{\pi}{6} \Rightarrow 2 = \frac{2}{3}\sin 3\times\frac{\pi}{6} + c \Rightarrow \frac{4}{3}$ | dM1 |
| | $s = \frac{2}{3}\sin 3\times\frac{\pi}{2} + \frac{4}{3} = \frac{2}{3}$ (m) | ddM1A1<br>[4] |
| | **ALT** | |
| | $s = 2 + \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \cos 3t\ \mathrm{d}t = 2 + \left[2\times\frac{1}{3}\sin 3t\right]_{\frac{\pi}{6}}^{\frac{\pi}{2}}$ | [M1dM1 |
| | $= 2 + \frac{2}{3}\left(\sin\frac{3\pi}{2} - \sin\frac{3\pi}{6}\right) = \frac{2}{3}$ | M1A1] |
| | **Total** | **9 marks** |

| Part | Mark | Notes |
|---|---|---|
| (a) | M1 | Sets $v = 1$ and solves for $\cos t$ to find a value for $t$<br>Accept an answer in degrees for this mark |
| | A1 | For the correct value for $t$ only in radians |
| (b) | M1 | For differentiating the given expression for $v$ to find acceleration.<br>Minimally acceptable derivative is $-k\sin 3t$ where $k \neq 0$ or 1 |
| | dM1 | For using the value of $t$ given to find a value for $a$ |
| | A1 | For the correct value of $a$ |
| (c) | M1 | For integrating the given expression for $v$ to find displacement (+ $c$)<br>Minimally acceptable integration is $\frac{l}{3}\sin 3t \quad l \neq 0, 1$ |
| | dM1 | For finding the value of the constant of integration $c$ |
| | ddM1 | For substituting the given value of $t$ to find a value for $s$<br>Note, this mark is dependent on both of the previous method marks being awarded. |
| | A1 | For the correct value of $s$ |
| | **ALT** | |
| | M1 | For integrating the given expression for $v$ to find displacement (+ $c$)<br>Minimally acceptable integration is $\frac{l}{3}\sin 3t \quad l \neq 0, 1$ |
| | dM1 | For a correct statement using when $t = \frac{\pi}{6}$, $x = 2$ |
| | ddM1 | For evaluating their limits – allow substitution either way around. |
| | A1 | For the correct value of $s$ |

| Question number | Scheme | Marks |
|---|---|---|
| 6 (a) | $XA=\dfrac{AM}{\cos 30^{\circ}}=\dfrac{x/2}{\sqrt{3}/2}=\dfrac{x}{\sqrt{3}}$ | M1A1 |
| | $DX=\sqrt{x^2-\left(\dfrac{x}{\sqrt{3}}\right)^2}=\sqrt{\dfrac{2}{3}}x,=\dfrac{\sqrt{2}}{\sqrt{3}}x\times\dfrac{\sqrt{3}}{\sqrt{3}}=\dfrac{\sqrt{6}}{3}x$ * | M1A1,A1 cso [5] |
| | **ALT** | |
| | $CM=\sqrt{x^2-\left(\dfrac{x}{2}\right)^2}=\dfrac{\sqrt{3}}{2}x$ e.g. $XM=\dfrac{1}{3}\times\dfrac{\sqrt{3}}{2}x=\dfrac{\sqrt{3}}{6}$ | [M1A1 |
| | $DX=\sqrt{CM^2-XM^2}=\sqrt{\left(\dfrac{\sqrt{3}}{2}x\right)^2-\left(\dfrac{\sqrt{3}}{6}x\right)^2}=\dfrac{\sqrt{6}}{3}x$ * | M1A1A1 cso] |
| (b) | $CM=\sqrt{x^2-\left(\dfrac{x}{2}\right)^2}=\dfrac{\sqrt{3}}{2}x$ | M1A1 |
| | $\cos\theta=\dfrac{\left(\dfrac{\sqrt{3}}{2}x\right)^2+\left(\dfrac{\sqrt{3}}{2}x\right)^2-x^2}{2\times\left(\dfrac{\sqrt{3}}{2}x\right)\times\left(\dfrac{\sqrt{3}}{2}x\right)}=\dfrac{1}{3}$ * | M1A1 cso [4] |
| (c) | Area of triangle $ABC=\dfrac{1}{2}\times x\times\dfrac{\sqrt{3}}{2}x=\left[\dfrac{\sqrt{3}}{4}x^2\right]$ | M1A1 |
| | Volume of pyramid $=\dfrac{1}{3}\times\dfrac{\sqrt{3}}{4}x^2\times\dfrac{\sqrt{6}}{3}x=\dfrac{16\sqrt{2}}{3}$ <br> $\Rightarrow x^3=64\Rightarrow x=4$ | M1A1 [4] |
| | **Total** | **13 marks** |

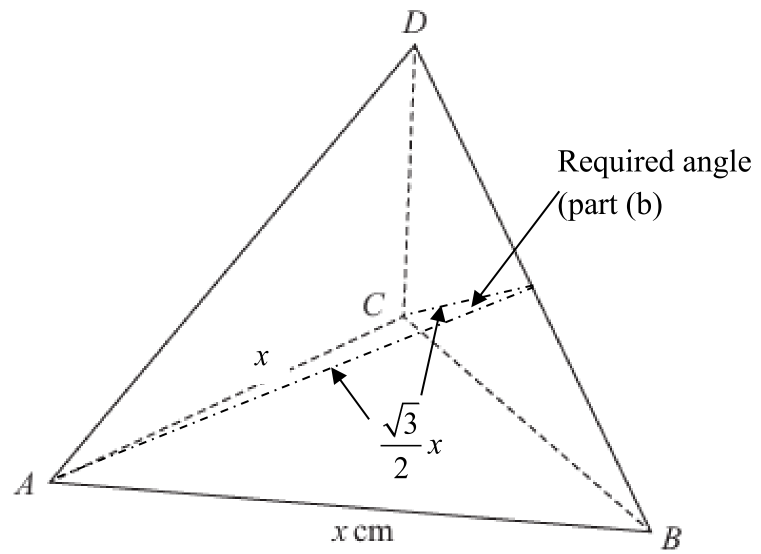
**Useful sketch (a)**



| Part | Mark | Notes |
|---|---|---|
| (a) | M1 | **Please mark parts a and b together on this question.** For using any trigonometry to find the length from one of the vertices of the triangle to the centre point below $D$ |
| | A1 | For the correct length. |
| | M1 | For using any correct trigonometry or Pythagoras theorem to find the required length. |
| | A1 | For the correct length in any form |
| | A1* cso | For the correct length in the required form |
| | **ALT** | |
| | M1 | For any appropriate method to find the length $CM$ **and** $XM$ |
| | A1 | For both correct lengths |
| | M1 | For any correct trigonometry or Pythagoras to find the required length. |
| | A1 | For the correct length in any form. |
| | A1* | For the correct length in the required form. |
| (b) | M1 | For using any appropriate method to find the length of a vertex to the opposite perpendicular. |
| | A1 | For the correct length [of $CM$] |
| | M1 | For using cosine rule or any other other appropriate method to find the required value of cos $\theta$ |
| | A1* cso | For the correct value as shown with no errors seen. |
| (c) | M1 | For finding the area of triangle $ABC$ |
| | A1 | For the correct area. |
| | M1 | For using the volume of the pyramid to find $x$ |
| | A1 | For $x = 4$ |

**Useful Sketch (b)**



| Question number | Scheme | Marks |
|---|---|---|
| 7 (a)(i) | $(1-3x)^{-\frac{1}{2}}=1+\left(-\frac{1}{2}\right)(-3x)+\frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)(-3x)^2}{2!}+\frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)(-3x)^3}{3!}$ | M1 |
| | $\Rightarrow(1-3x)^{-\frac{1}{2}}=1+\frac{3}{2}x+\frac{27}{8}x^2+\frac{135}{16}x^3+\ldots$ | A1A1<br>(3) |
| (ii) | $-\frac{1}{3}\le x<\frac{1}{3}$ accept $\lvert x\rvert<\frac{1}{3}$ and $-\frac{1}{3}<x<\frac{1}{3}$ | B1<br>(1) |
| (b) | $(2-x)\left(1+\frac{3}{2}x+\frac{27}{8}x^2+\frac{135}{16}x^3\right)=2+2x+\frac{21}{4}x^2+\frac{27}{2}x^3$ | M1A1ftA1<br>(3) |
| (c) | $\int_0^{0.1}\frac{(2-x)}{\sqrt{(1-3x)}}\,\mathrm{d}x=\int_0^{0.1}2+2x+\frac{21}{4}x^2+\frac{27}{2}x^3\,\mathrm{d}x=\left[2x+\frac{2x^2}{2}+\frac{21}{12}x^3+\frac{27}{8}x^4\right]_0^{0.1}$ | M1A1ft |
| | $=(0.2+0.01+0.00175+0.0003375)=0.2120875\approx0.2121\ (4\text{ dp})$ | M1A1<br>(4) |
| | **Total** | **11 marks** |

| Part | Mark | Notes |
|---|---|---|
| (a)(i) | M1 | Applies Binomial theorem correctly with powers of $x$ correct and the denominators correct.<br>The expansion must begin with 1<br><br>Minimally acceptable expansion is as follows:<br>$(1+3x)^{-\frac{1}{2}}=1\pm\alpha x\pm\frac{\beta x^2}{2!}\pm\frac{\delta x^3}{3!}$ where $\alpha,\beta,\delta$ are non zero constants. |
| | A1 | At least one term involving a power of $x$, correctly simplified. |
| | A1 | Fully correct |
| (ii) | B1 | For the correct range |
| (b) | M1 | For attempting to multiply their expansion by $(2-x)$ |
| | A1 | For at least two terms correct, ft their values. |
| | A1 | Fully correct |
| (c) | M1 | Attempts to integrate their expression from (b) which must contain at least three terms. [See General Guidance for the definition of an attempt] |
| | A1ft | For a correct integration, allow a ft here. Please check their integration. |
| | M1 | For a correct explicit substitution of the given values. You do not need to see 0 substituted. |
| | A1 | For the correct value, rounded correctly.<br>Note: Calculator value is 0.2121805….. |

| Question number | Scheme | Marks |
|---|---|---|
| 8 | $\tan 30 = \frac{r}{h} \Rightarrow r = h\tan 30^{\circ} = \left(\frac{h}{\sqrt{3}}\right)$ | M1 |
| | $V = \frac{1}{3}\pi r^2 h \Rightarrow V = \frac{1}{3}\pi\left(\frac{h}{\sqrt{3}}\right)^2 h = \left(\frac{1}{9}\pi h^3\right)$ | M1 |
| | $\frac{dV}{dh} = \frac{1}{3}\pi h^2$ | M1 |
| | $\frac{dh}{dt} = 0.04$ | B1 |
| | $\frac{dh}{dt} = \frac{dV}{dt}\times\frac{dh}{dV} \Rightarrow 0.04 = \frac{dV}{dt}\times\frac{1}{\frac{1}{3}\pi\times 2^2} \Rightarrow \frac{dV}{dt} = 0.04\times\frac{1}{3}\pi\times 2^2 = \frac{4}{75}\pi$ | M1A1 |
| | | [6] |
| | **Total** | **6 marks** |

| Mark | Notes |
|---|---|
| M1 | For finding an expression for $h$ using tan ratio correctly. |
| M1 | For finding the volume using a **correct** formula for the volume, in terms of $h$ only using their expression for $r = kh$ |
| M1 | For differentiating their $V$ provided it is in terms of $h$ only. The differentiation of their expression must be correct for this mark. Please check the differentiation. |
| B1 | For stating or using $\frac{dh}{dt} = 0.04$ and seen anywhere<br>**This is an M1 in epen**. |
| M1 | For substituting their values<br>into a correct chain rule, in any form, in order to find a value for $\frac{dV}{dt}$ |
| A1 | For the correct **exact** value of $\frac{dV}{dt}$ |

| Question number | Scheme | Marks |
|---|---|---|
| 9 (a) | $\tan(A-B)=\dfrac{\sin(A-B)}{\cos(A-B)}=,\dfrac{\sin A\cos B-s\text{in }B\cos A}{\cos A\cos B+\sin A\sin B}$ | M1,M1 |
| | $=\dfrac{\dfrac{\sin A\cos B-s\text{in }B\cos A}{\cos A\cos B}}{\dfrac{\cos A\cos B+\sin A\sin B}{\cos A\cos B}}$ | M1 |
| | $=\dfrac{\dfrac{\sin A}{\cos A}-\dfrac{\sin B}{\cos B}}{1+\dfrac{\sin A\sin B}{\cos A\cos B}}=\dfrac{\tan A-\tan B}{1+\tan A\tan B}$ * | A1 cso [4] |
| (b) | $2y+5x-7=0\Rightarrow\text{gradient}=-\dfrac{5}{2}$ | B1 |
| | $3y-2x+3=0\Rightarrow\text{gradient}=\dfrac{2}{3}$ | B1 |
| | $\tan\theta=\left\lvert\dfrac{\frac{2}{3}-\left(-\frac{5}{2}\right)}{1+\frac{2}{3}\times\left(-\frac{5}{2}\right)}\right\rvert=\left\lvert-\dfrac{19}{4}\right\rvert$ or $\tan\theta=\left\lvert\dfrac{\left(-\frac{5}{2}\right)-\frac{2}{3}}{1+\frac{2}{3}\times\left(-\frac{5}{2}\right)}\right\rvert=\dfrac{19}{4}$ | M1A1 |
| | $\theta=180^{\circ}-78.11134^{\circ}\approx102^{\circ}$ | B1 [5] |
| **ALT 1** | Intercept on axes of line $2y+5x-7=0\Rightarrow y=\dfrac{7}{2},x=\dfrac{5}{2}$ | B1 |
| | Intercept on axes of line $3y-2x+3=0=0\Rightarrow y=-1,x=\dfrac{3}{2}$ | B1 |
| | $2y+5x-7=0\Rightarrow\tan\theta=\dfrac{\frac{7}{5}}{\frac{7}{2}}\Rightarrow\theta=21.80^{\circ}$ <br> OR $\tan\theta'=\dfrac{\frac{7}{2}}{\frac{7}{5}}\Rightarrow\theta'=68.2^{\circ}$ | M1 |
| | $3y-2x+3=0\Rightarrow\tan\alpha=\dfrac{\frac{3}{2}}{1}\Rightarrow\alpha=52.31^{\circ}$ <br> OR $\tan\alpha'=\dfrac{1}{\frac{3}{2}}\Rightarrow\alpha'=33.7^{\circ}$ | A1 |
| | Required angle = $180^{\circ}-21.80^{\circ}-56.31^{\circ}=102^{\circ}$ <br> OR $68.2^{\circ}+33.7^{\circ}=102^{\circ}$ | B1 [5] |

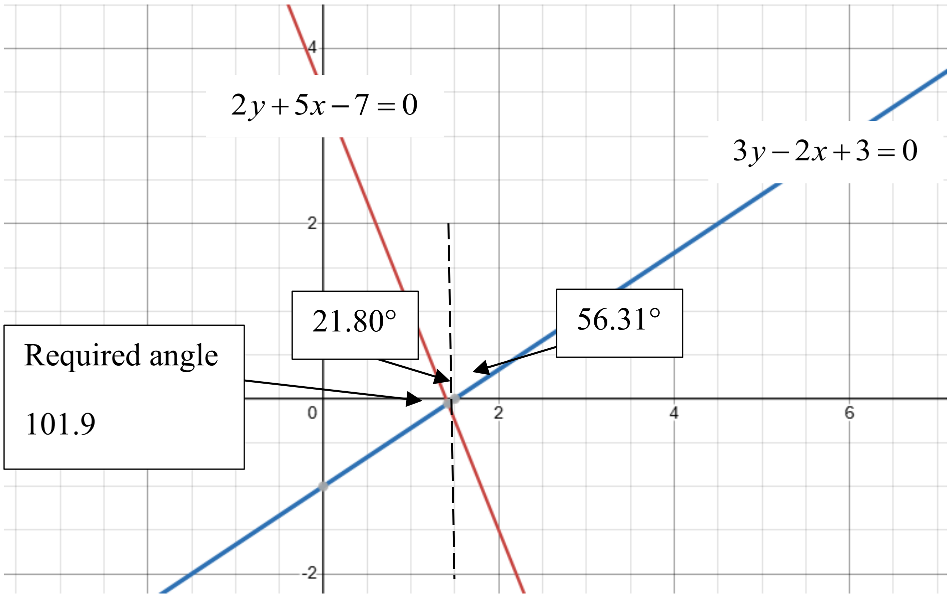
| | | |
|---|---|---|
| **ALT 2** | $y$ intercepts of both lines on $y$-axes $\frac{7}{2}$ and $-1$ | B1 |
| | Coordinates of the intersection of both lines $\left(\frac{27}{19},-\frac{1}{19}\right)$ | B1 |
| | Length of one side of the triangle $\sqrt{\left(\frac{27}{19}\right)^2+\left(\frac{135}{38}\right)^2}=3.826$ | M1 |
| | Length of other side of triangle $\sqrt{\left(\frac{27}{19}\right)^2+\left(\frac{18}{19}\right)^2}=1.708$ or $\frac{9\sqrt{13}}{19}$ | A1 |
| | Required angle = $\cos^{-1}\left(\frac{3.826^2+1.708^2-4.5^2}{2\times3.826\times1.708}\right)=102^\circ$ | B1 |
| **ALT 3** | $x$ intercepts of both lines on $x$-axes | [B1 |
| | Coordinates of the intersection of both lines $\left(\frac{27}{19},-\frac{1}{19}\right)$ | B1 |
| | Length of one side of the triangle = $\frac{\sqrt{29}}{35}$ | M1 |
| | Length of other side of triangle $\frac{\sqrt{13}}{38}$ | A1 |
| | Required angle = awrt $102^\circ$ | A1] |
| | **Total** | **9 marks** |

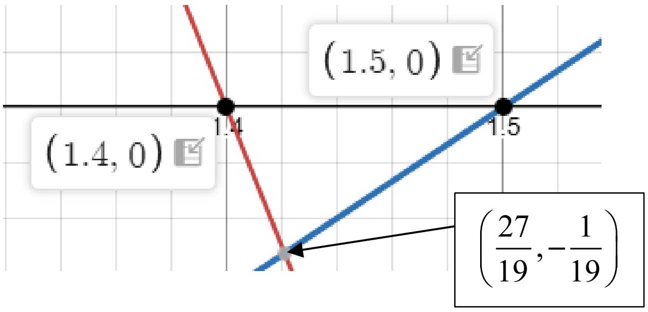
| Part | Mark | Notes |
|---|---|---|
| (a) | M1 | Uses the identity for $\tan=\frac{\sin}{\cos}$ |
| | M1 | Uses the identities for $\cos(A-B)$ and $\sin(A-B)$ correctly |
| | M1 | Divides through by $\cos A\cos B$ |
| | A1* cso | For the given result with no errors. We need to see either on the LHS or RHS of their working $\tan(A-B)=...$ for the award of this mark. |
| (b) | In this part of the question candidates will mix and match methods.<br>• Please score marks as soon as they are seen, irrespective of the method they appear to be using.<br>• Please check any diagrams they draw for evidence of intercepts.<br>• For the final answer accept awrt $102^\circ$ | |
| | B1 | For one **correct** gradient |
| | B1 | For both **correct** gradients |
| | M1 | For applying the given addition formula for tan **correctly** |
| | A1 | For the **correct** value of $\tan\theta$ |
| | B1 | For finding the obtuse angle from their value of tan. |
| | **In ALT 1 and ALT 2 values used must be correct throughout.** | |

| | | |
|---|---|---|
| **ALT 1** | B1 | Finds coordinates of intersection of one line with axes |
| | B1 | Finds coordinates of intersection of both lines with axes |
| | M1 | Finds one vertical or horizontal angle for one line |
| | A1 | Finds both vertical or horizontal angles |
| | B1 | For the correct obtuse angle between the two lines |
| | | |
| **ALT 2** | B1 | Finds the intersection of both lines with the $y$-axis |
| | B1 | Finds the coordinates of the point of intersection between the two lines |
| | M1 | Uses Pythagoras to find the length of one line |
| | A1 | Uses Pythagoras theorem to find the angle of bothe lines |
| | B1 | Uses cosine rule to find the correct value for the obtuse angle. |
| | | |
| **ALT 3** | B1 | Finds the intersection of both lines with the $x$-axis |
| | B1 | Finds the coordinates of the point of intersection between the two lines |
| | M1 | Uses Pythagoras theorem to find the length of one of the sides of the triangle |
| | A1 | Uses Pythagoras theorem to find the length of both of the sides of the triangle |
| | A1 | For the required angle of awrt 102° |

**If you see any methods based on vectors please send to Review**

**<u>Useful Sketchs Part (b)</u>**



| Question number | Scheme | Marks |
|---|---|---|
| 10 | $3\log_3 x^2 - 4\log_x 3 = 12 \Rightarrow 6\log_3 x, -4\dfrac{\log_3 3}{\log_3 x} - 12 = 0$ | M1,M1 |
| | $\Rightarrow 6(\log_3 x)^2 - 12(\log_3 x) - 4 = 0$ | M1 |
| | $\Rightarrow \log_3 x = \dfrac{--12 \pm \sqrt{(-12)^2 - 4\times 6\times -4}}{2\times 6} = \dfrac{3\pm\sqrt{15}}{3}$ | M1 |
| | $\Rightarrow \log_3 x = 2.29099..., -0.29099....$ | A1 |
| | $x = 3^{2.29099} = 12.390... \approx 12.4 \qquad x = 3^{-0.29099} = 0.72637... \approx 0.726$ | M1A1 [7] |
| | **ALT** works in base $x$ | |
| | $3\log_3 x^2 - 4\log_x 3 = 12, \Rightarrow \dfrac{6\log_x x}{\log_x 3}, -4\log_x 3 - 12 = 0$ | [M1,M1 |
| | $\Rightarrow 4(\log_x 3)^2 + 12(\log_x 3) - 6 = 0$ | M1 |
| | $\Rightarrow \log_3 x = \dfrac{-12 \pm \sqrt{(12)^2 - 4\times -6\times 4}}{2\times 4} = \dfrac{-3\pm\sqrt{15}}{2}$ | M1 |
| | $\Rightarrow \log_3 x = 0.43649...., -3.43649....$ | A1 |
| | $3 = x^{0.43649} \Rightarrow \sqrt[0.43649]{3} = 12.39....., \quad 3 = x^{-3.43649} \Rightarrow \sqrt[-3.43649]{3} = 0.726...$ | M1A1] |
| | **Total** | **7 marks** |

| Mark | Notes |
|---|---|
| **These general notes cover both methods in base 3 or base *x*** | |
| M1 | For dealing with the power law. Note: $3\log_3 x^2 = 6\log_3 x$ the only acceptable approach here. If you see an equation in $\log_3 x^6$ leading to a correct answer, pleaser send to Review. |
| M1 | For changing a base of logs |
| M1 | For forming a 3TQ which can be in any form.<br>i.e., It does not have to be all on one side = 0<br>e.g., accept $6(\log_3 x)^2 = 12(\log_3 x) + 4$ |
| M1 | Solves their 3TQ using an explicit correct method. Do not accept calculator solutions only. |
| A1 | For both correct values |
| M1 | Undoes the log for at least one of their values. You need to see evidence of the method and this step must be seen . Do not accept solutions appearing out of nowhere. |
| A1 | For both correct values.<br>Accept awrt 12.4 and 0.726 |

| Question number | Scheme | Marks |
|---|---|---|
| 11 (a) | $8a+4ar=8$ | B1 |
| | $2\times a\times ar^2=ar\Rightarrow\left[ar=\dfrac{1}{2}\right]$ | B1 |
| | Solves simultaneous equations | |
| (i) | $8a+4\times\dfrac{1}{2}=8\Rightarrow a=\dfrac{6}{8}=\left(\dfrac{3}{4}\right)$ | |
| (ii) | $\dfrac{3}{4}\times r=\dfrac{1}{2}\Rightarrow r=\dfrac{2}{3}$ | M1A1 [4] |
| (b) | $S_\infty=\dfrac{\frac{3}{4}}{1-\frac{2}{3}}=\dfrac{9}{4}$ | M1A1 [2] |
| (c) | $U_n=\dfrac{3}{4}\left(\dfrac{2}{3}\right)^{n-1}\Rightarrow\dfrac{3}{4}\left(\dfrac{2}{3}\right)^{n-1}<0.06$ | M1 |
| | $\left(\dfrac{2}{3}\right)^{n-1}<\dfrac{2}{25}$ | M1 |
| | $n-1>\dfrac{\lg\left(\frac{2}{25}\right)}{\lg\left(\frac{2}{3}\right)}$ | M1 |
| | $n>\dfrac{\lg\left(\frac{2}{25}\right)}{\lg\left(\frac{2}{3}\right)}+1\Rightarrow n>7.229........$ | M1 |
| | $n=8$ | A1 [5] |
| | **Total** | **11 marks** |

| Part | Mark | Notes |
| --- | --- | --- |
| (a) | B1 | For one correct equation. |
| | B1 | For both correct equations. |
| | M1 | For any method to solve the simultaneous equations. |
| | A1 | For correct values of $a$ and r |
| (b) | M1 | Uses the correct formula for the sum to infinity with their values.<br>Use of a value of $r > 1$ is M0 |
| | A1 | For the correct value |
| (c) | **In this part of the question accept any inequality or equals sign for the M marks but in order to gain the final A mark the inequality work must be correct throughout including the change of direction of the inequality.**<br><br>**If the candidate works with the sum of a Geometric Series, the maximum mark obtainable is M0M0M0M1A0** | |
| | M1 | For using the correct $n$th term of a Geometric series with their $a$ and $r$ with the given value of 0.06 |
| | M1 | Simplifies to the next stage and obtains, for example, $\left(\frac{2}{3}\right)^{n-1} < 0.08$ oe |
| | M1 | Rearranges to find an expression for either $n - 1$ or $n$ using logs |
| | M1 | Evaluates the logs correctly to obtain a value for $n$ |
| | A1 | For $n = 8$<br>**Note all inequality work must be correct for this mark to be awarded**. |